

First name .....

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Degree program name .....

## Exercise 145: Resonance frequency in LRC Circuits

Table: The resistor voltage as a function of the current frequency  $f$  in parallel and series RLC circuits.

C = .....  $\mu\text{F}$ ,      L = ..... H,      R = .....  $\Omega$

| $f$ [Hz] | $V_{Series}$ [V] | $I_{Series}$ [mA] | $V_{Parallel}$ [V] | $I_{Parallel}$ [mA] |
|----------|------------------|-------------------|--------------------|---------------------|
| 30       |                  |                   |                    |                     |
| 50       |                  |                   |                    |                     |
| 60       |                  |                   |                    |                     |
| 70       |                  |                   |                    |                     |
| 75       |                  |                   |                    |                     |
| 80       |                  |                   |                    |                     |
| 85       |                  |                   |                    |                     |
| 88       |                  |                   |                    |                     |
| 90       |                  |                   |                    |                     |
| 93       |                  |                   |                    |                     |
| 95       |                  |                   |                    |                     |
| 100      |                  |                   |                    |                     |
| 105      |                  |                   |                    |                     |
| 110      |                  |                   |                    |                     |
| 120      |                  |                   |                    |                     |
| 130      |                  |                   |                    |                     |
| 150      |                  |                   |                    |                     |

Complementary measurements.

|   |  |
|---|--|
| Determined resonant frequency $f_r$ , [Hz]  |  |
| Determined resonant angular frequency $\omega_r$ , [ $\text{s}^{-1}$ ]                      |  |
| Theoretical resonant angular frequency $\omega_r^{theor.}$ , [ $\text{s}^{-1}$ ]            |  |
| Percent difference: $Bp = \frac{ \omega_{res} - \omega_{res}^t }{\omega_{res}} \cdot 100\%$ |  |

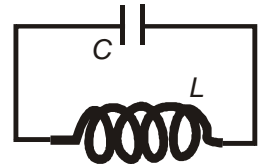
## Exercise 145: Resonance frequency in LRC Circuits

### PURPOSE

This laboratory experiment is designed to explore resonance phenomena within a resistor-inductor-capacitor (RLC) circuit, both in series and parallel configurations. The objective is to analyze how the current amplitude in the circuit varies with the frequency of the applied voltage.

### THEORY

In an electrical LC (inductor-capacitor) circuit, the characteristics of charge, current, and potential differ from exponential decay, instead undergoing sinusoidal variations with an angular frequency denoted as  $\omega$ . These oscillations result in the generation of electromagnetic oscillations in both the electric field of the capacitor (C) and the magnetic field of the inductor (L)



When a charged capacitor, with a capacitance of  $C$ , is connected to an inductor possessing inductance denoted as  $L$ , it initiates a discharge process. As the charge within the capacitor diminishes, the energy stored in the electric field of the capacitor similarly diminishes. This energy is transferred to the magnetic field of the inductor because the discharge current from the capacitor flows through the inductor. Consequently, due to the electromotive force generated by the inductor, a current is induced in the opposite direction, causing the capacitor to commence charging. The charging process of the capacitor is in inverse proportion to the discharge, gradually restoring the circuit to its initial state. This cycle repeats continuously with minimal energy loss, particularly in cases where circuit resistance is either nonexistent or negligible.

### Resonance in electric circuits

In an AC circuit comprising resistors, capacitors, and inductors connected either in series or in parallel, resonance can be observed at a specific frequency known as the resonant frequency. This phenomenon manifests when the inductive reactance matches the capacitive reactance at a particular frequency. While this explanation is somewhat simplified, it serves the intended purpose for this exercise.

### Capacitive reactance

Let apply to the capacitor an alternating voltage:  $U(t) = U_0 \sin(\omega t)$ . The current flowing through the circuit with the capacitor equals:

$$I_C(t) = C \frac{dU}{dt} = C\omega U_0 \cos(\omega t) = I_{C0} \cos(\omega t) = I_{C0} \sin\left(\omega t + \frac{\pi}{2}\right).$$

It means that we have a current (with amplitude  $I_{C0} = C\omega U_0$ ) that leads the input voltage in phase by  $90^\circ$  (the cosine function is shifted with respect to the sine function by  $\frac{\pi}{2} \text{ rad} = 90^\circ$ ). Herein, “leads” means that the current peaks is before the voltage peak at the capacitor. Comparing  $I_{C0} = \frac{U_0}{1/C\omega}$  with Ohm's law  $I = \frac{U}{R}$ , we get that the resistance of the capacitor depends on the frequency of the current; therefore, we call it capacitive reactance and mark it by  $X_C = \frac{1}{C\omega}$ .

**Inductive reactance**

Similarly, let apply to the inductor the voltage:  $U(t) = U_0 \sin(\omega t)$ . Because  $U(t) = L \frac{dI}{dt}$ , we can write that the current flowing through the inductor equals:

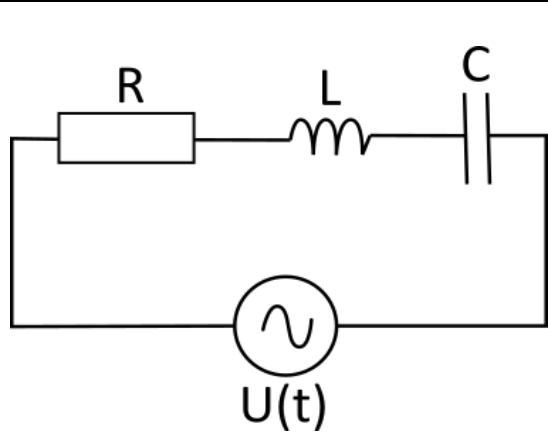
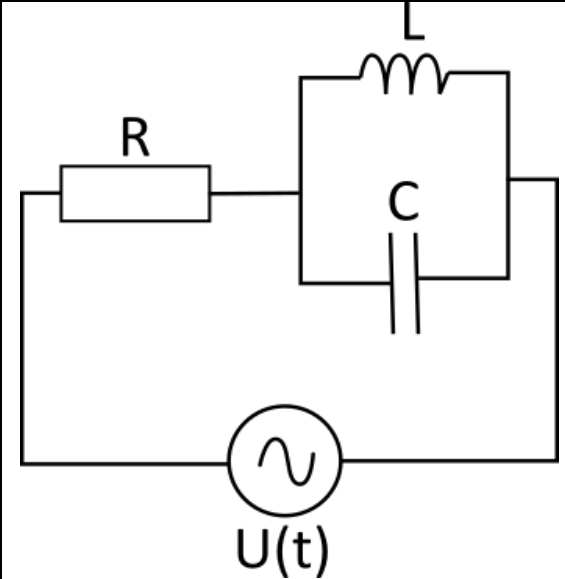
$$I_L(t) = \int dI_L = \frac{U_0}{L} \int \sin(\omega t) dt = -\frac{U_0}{L\omega} \cos(\omega t) = \frac{U_0}{L\omega} \sin\left(\omega t - \frac{\pi}{2}\right) = I_{L0} \sin\left(\omega t - \frac{\pi}{2}\right).$$

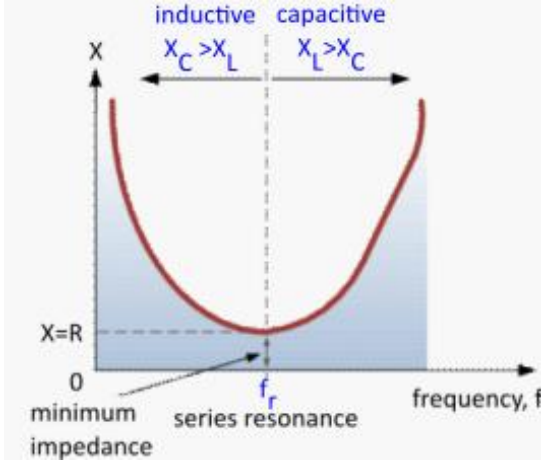
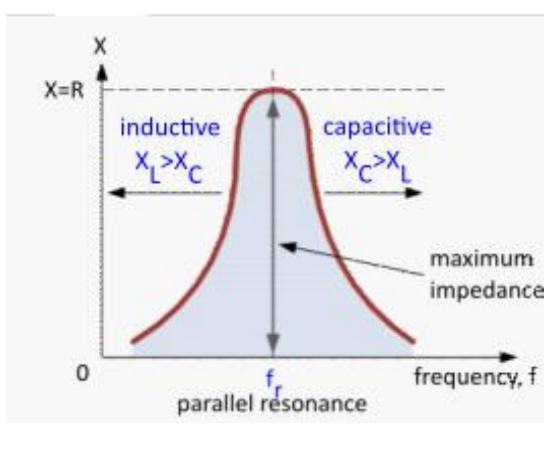
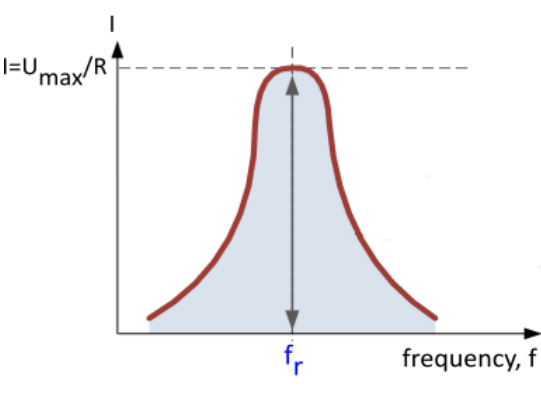
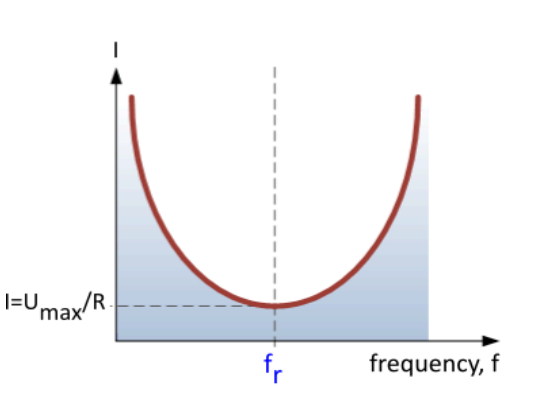
For an inductive load the current lags the voltage in phase by  $90^\circ$ . Herein, “lags” means that the current peaks is after the voltage peak at the inductor. Comparing  $I_{L0} = \frac{U_0}{L\omega}$  with Ohm's law  $I = \frac{U}{R}$ , we get that the resistance of the inductor depends on the frequency of the current; therefore, we call it inductive reactance and mark it by  $X_L = \omega L$

Table 1) Phase and Amplitude Relations for Alternating Currents and Voltages

| Circuit Element | Symbol | Resistance or Reactance   | Phase of the Current      | Phase constant (or angle) | Amplitude relation |
|-----------------|--------|---------------------------|---------------------------|---------------------------|--------------------|
| Resistor        | R      | R                         | In phase with $V_R$       | $0^\circ$                 | $V_R = I_R R$      |
| Capacitor       | C      | $X_C = \frac{1}{C\omega}$ | Leads $V_C$ by $90^\circ$ | $-90^\circ$               | $V_C = I_C X_C$    |
| Inductor        | L      | $X_L = \omega L$          | Lags $V_L$ by $90^\circ$  | $90^\circ$                | $V_L = I_L X_L$    |

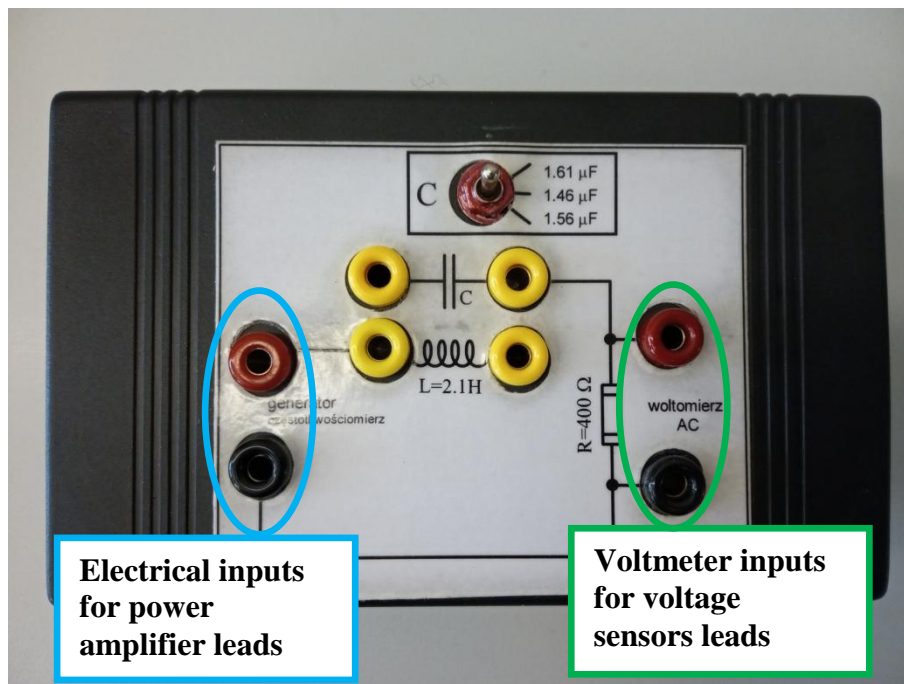
**Resonance in series and parallel RLC electric circuits**

|                                | Series RLC circuit  | Parallel RLC circuit   |
|--------------------------------|---|--|
| Circuit                        |  |  |
| Total impedance of the circuit | $X = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$                       | $X = \sqrt{R^2 + \left(\frac{1}{\omega C} - \frac{1}{\omega L}\right)^2}$            |

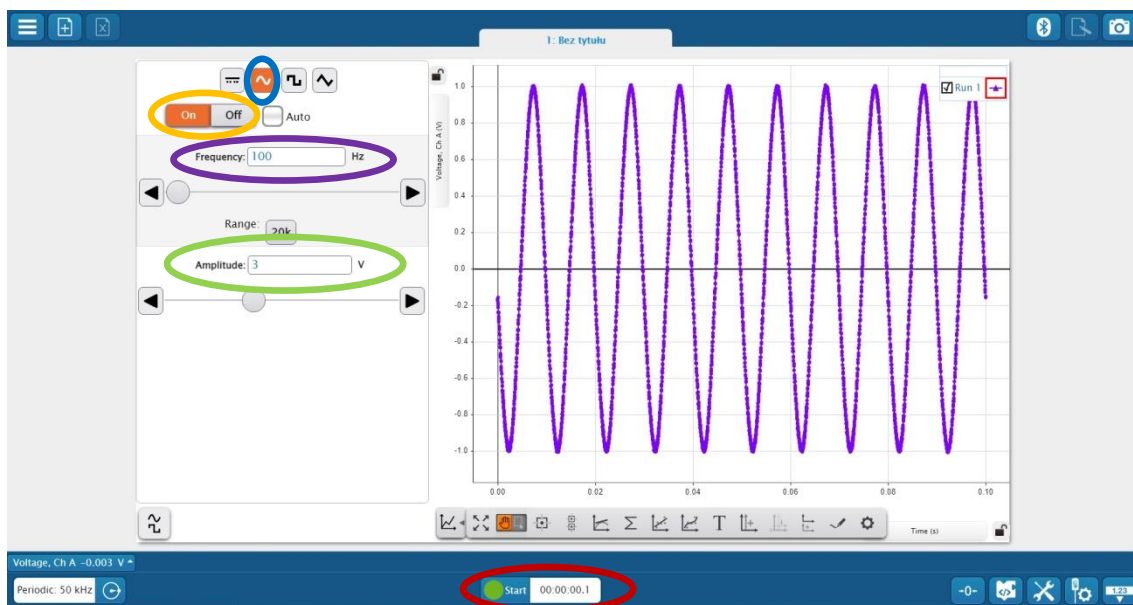
|   |   |  |
|---|---|--|
| <p>Total impedance of the circuit</p>       |    |    |
| <p>Resonance frequency</p>                  | $X_C = X_L$ $\frac{1}{\omega L} = \omega C$ $2\pi f = \omega_r = \frac{1}{\sqrt{LC}} \text{ yields } f_r = \frac{1}{2\pi\sqrt{LC}}$   |  |
| <p>Current flowing through the resistor</p> |   |   |
| <p>Features</p>                             | <ul style="list-style-type: none"> <li>• At a frequency below the resonant frequency, i.e. <math>f &lt; f_r</math>, <math>X_C &gt; X_L</math>. Hence the circuit is capacitive.</li> <li>• At a frequency above the resonant frequency, i.e. <math>f &gt; f_r</math>, <math>X_L &gt; X_C</math>. Hence the circuit is inductive.</li> <li>• At the resonant frequency, i.e. <math>f = f_r</math>, <math>X_L = X_C</math>, the current is maximum, and the impedance is minimum. In this state, the circuit can act as an acceptor circuit.</li> </ul> | <ul style="list-style-type: none"> <li>• At a frequency below the resonant frequency, i.e. <math>f &lt; f_r</math>, <math>X_L &gt; X_C</math>. Hence the circuit is inductive.</li> <li>• At a frequency above the resonant frequency, i.e. <math>f &gt; f_r</math>, <math>X_C &gt; X_L</math>. Hence the circuit is capacitive.</li> <li>• At the resonant frequency, i.e. <math>f = f_r</math>, <math>X_L = X_C</math>, the current is minimum, and the impedance is maximum. In this state, the circuit can act as a rejector circuit.</li> </ul> |

### Performance of the task

1. Turn on the power (turn the red "knob" in the direction of the arrows (it should pop out), turn the key, as in a car, and let go).
2. Turn on: (1) the PASCO universal interface and then (2) the computer and wait for Windows.

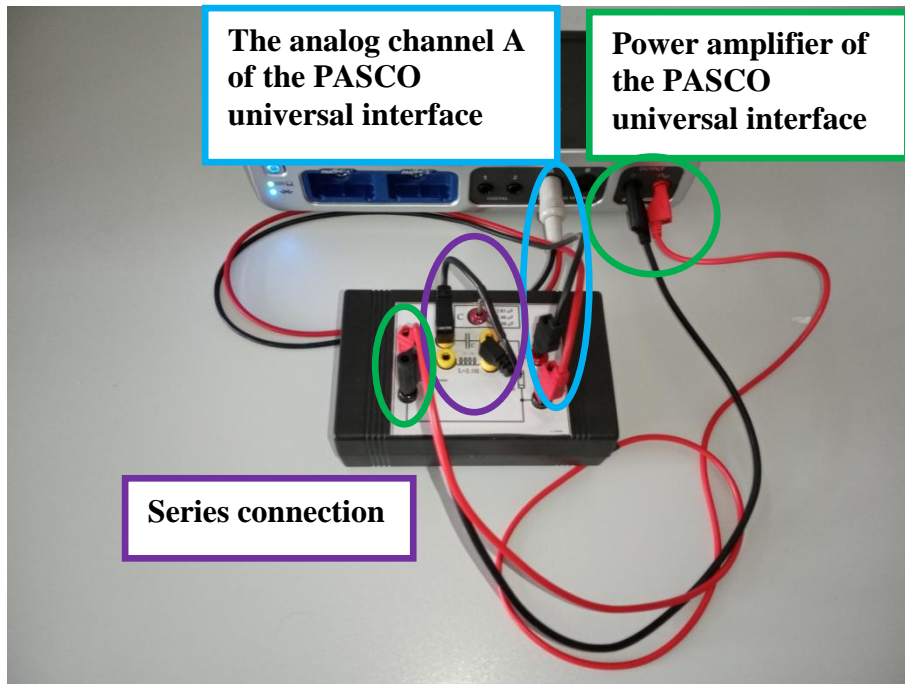


3. Connect the power amplifier leads (the part of the PASCO universal interface) to an appropriate electrical inputs called “generator częstotliwościomierz”.
4. Connect the voltage sensor leads (the analog channel A of the PASCO universal interface) to voltmeter inputs called (“woltmierz AC”).
5. Double-click on the **Ex145 application**.

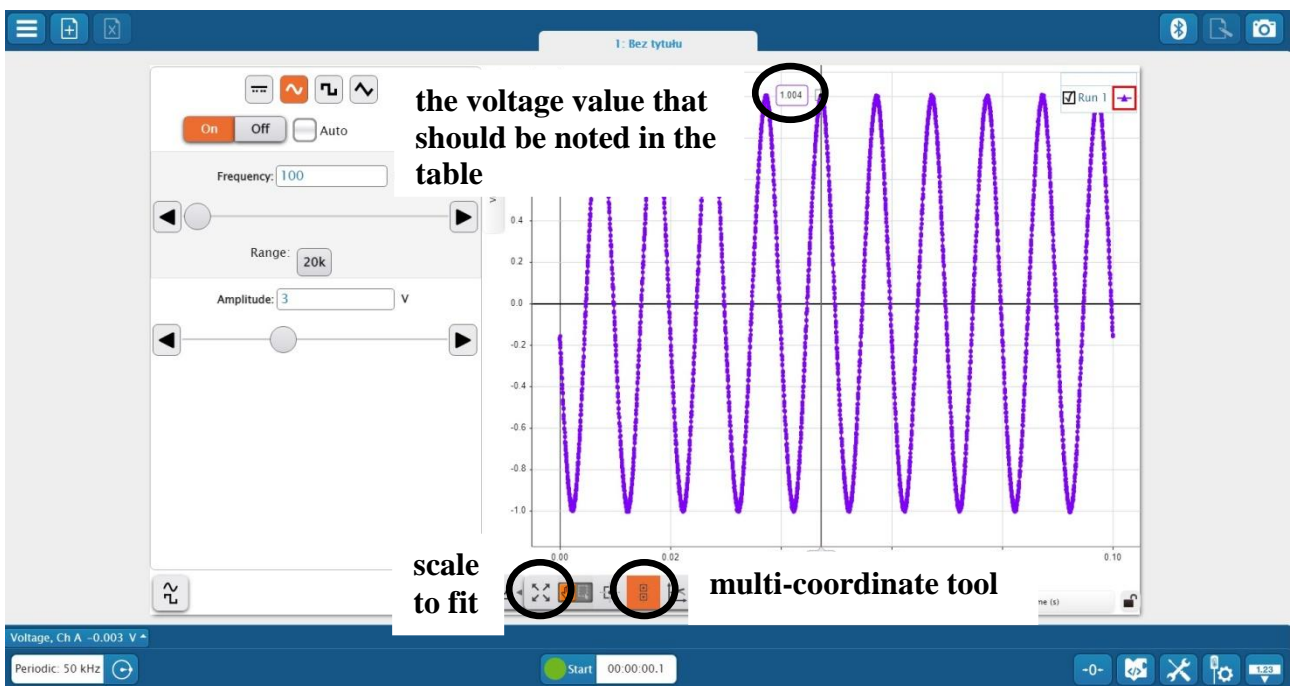




6. The Signal generator should be set to **amplitude 3 V** (please not exceed this value), **sine AC waveform**, at **frequency 30.00 Hz**. The signal generator is set to **ON** so it will start automatically when you click **Start**.

- Appropriately connect yellow banana plug-ins in an appropriate way to achieve a series connection between the capacitor and the inductor coil. The proper connections are marked on the bottom picture.

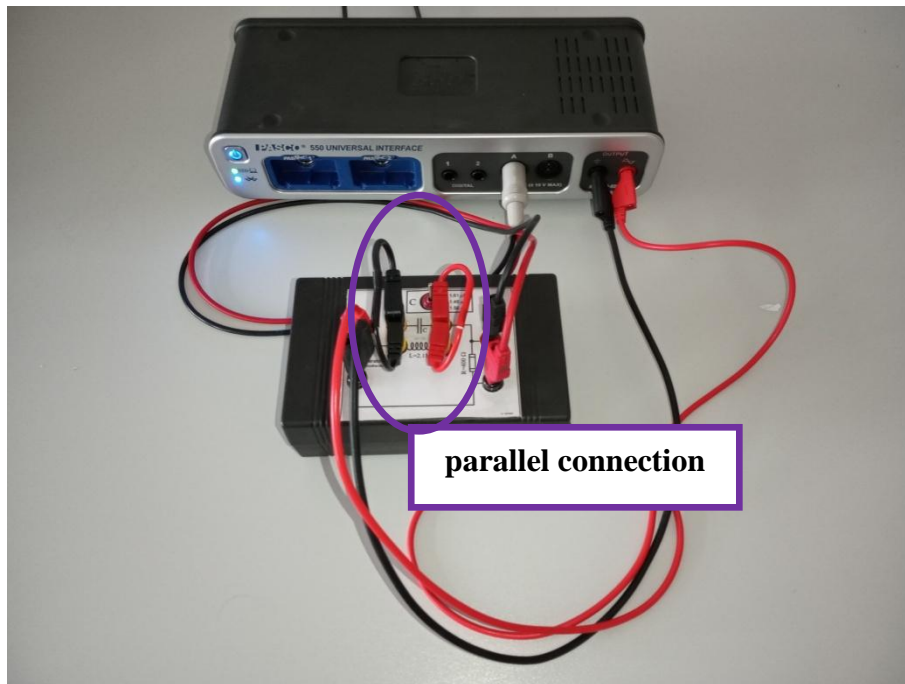


- Click the START button (in the application) to start monitoring data. The Signal Generator will start automatically, and data will also be collected automatically.

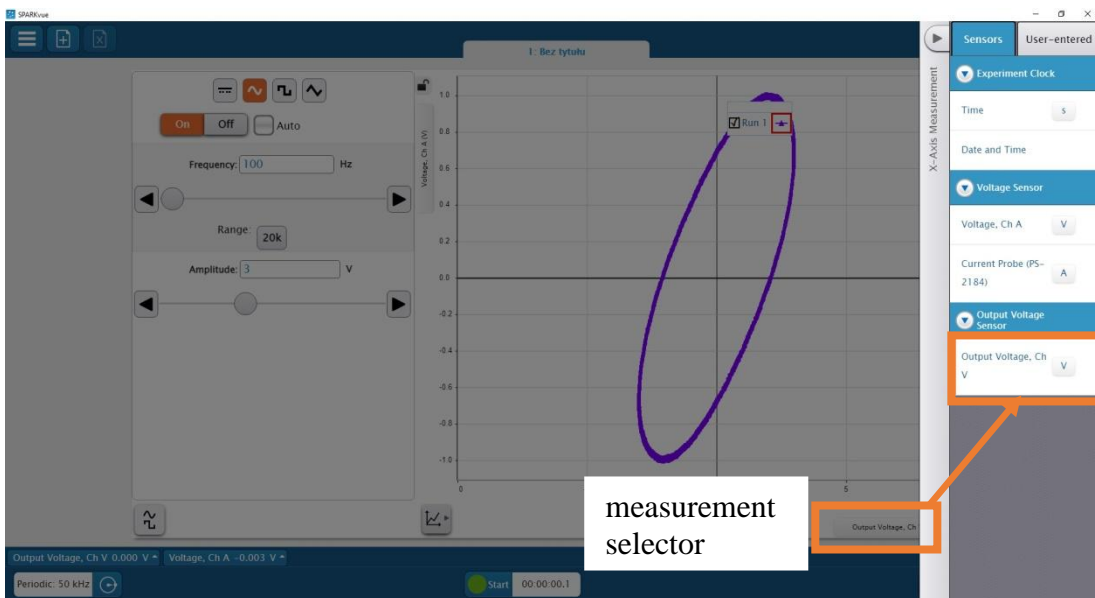


- Click on  - the scale to fit the recorded data into the graph window.
- Use  the multi-coordinate tool to read **the peak of the voltage** across the resistor in our RLC circuit. Note that value in the table.

11. In the signal generator window, change the frequency value to the next from the table.
12. Repeat steps 8-11 until the last frequency value in the table is reached.
13. Change the connection of yellow banana plug-ins to achieve the parallel connection between the capacitor and the inductor coil.



14. Repeat steps 8-11 to complete the table – note the peak of the voltage across the resistors for parallel connection of the capacitor and the inductor coil.
15. Change the connection of the yellow banana plug-ins to achieve the series connection between the capacitor and the inductor coil again. Set the generator's frequency to the frequency for which the measured voltage  $V_{Series}$  was highest for the series RLC circuit (according to the table).



16. Click the measurement selector shown above in the picture to change the measurement variable from time to voltage. In the measurement selection pane, select an Output Voltage, Ch V.

17. If voltages applied to the X and Y axes are shifted in phase, you will see an ellipse or a circle on the screen. If voltages are in phase, there will be a straight line on the screen. That means that the RLC circuit is in resonance (adjust the frequency changing it every 0.1 Hz to the resonant frequency  $f_r$ ). Note the frequency value in the table – determined resonance frequency.
18. Close the application and turn off the computer.

### ***Data analysis***

1. Draw the current-frequency  $I(f)$  relationship for a series and parallel circuit on a graph and mark the resonant frequency  $f_r$ .
2. Calculate the resonant angular frequency:  $\omega_r = 2\pi f_r$
3. Calculate the theoretical resonant angular frequency:  $\omega_r^{theor.} = \frac{1}{\sqrt{LC}}$
4. Calculate the percentage difference between the theoretical and experimental values of the resonant angular frequency.

### ***Questions for discussion***

In your conclusions, analyze which resonant frequency, experimental or theoretical, you determined more accurately?

Whether the resonance frequency is the same for parallel and series RLC circuits? Why?