

First name .....

Date .....

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Degree program name .....

## Exercise 133: The Interference of Acoustic Waves - Beats

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Table I. Determination of the beats frequencies

Measurement	Time, [ms]		Difference, [ms]	Number of cycles	Frequency of generator, [Hz]	
	$t_1$	$t_2$			$t_2 - t_1$	$N$
No						
1						
2						
3						

Table II. Theoretical the beats frequency for a given measurement

Measurement	Calculated frequency beats, [Hz]	Theoretical frequency beats, [Hz]	Percentage difference, [%]
No	$f_d = \frac{N}{t_2 - t_1}$	$f_t =  f_1 - f_2 $	$\frac{ f_t - f_d }{f_t} \cdot 100\%$
1			
2			
3			

## Exercise 133: The Interference of Acoustic Waves - Beats

### **PURPOSE**

This exercise entails the analysis of beating wave frequencies, contingent upon the choice of frequencies for two intersecting acoustic waves. The outcome of this phenomenon, known as superposition or interference, becomes evident on the monitor screen within the Interference-Beats window of the primary program. This window functions akin to an oscilloscope display.

### **THEORY**

When two sound waves, each differing slightly in frequency, overlap at a specific point, they generate a resultant vibration among the molecules of the medium. This resultant vibration exhibits the same frequency as the component waves but varies in amplitude over time, giving rise to what we refer to as a "beats". The frequency of this resultant wave can be calculated as the arithmetic mean of the frequencies of the overlapping waves. Moreover, the frequency of the amplitude modulation, known as the beats frequency (denoted as  $f_d$ ), is equivalent to the difference between the frequencies  $f_1$  i  $f_2$  of the component waves:

$$f_d = |f_1 - f_2|.$$

To illustrate these concepts, we will provide a demonstration using the interference of harmonic waves.

#### **Harmonic Vibrations:**

Cyclic motion, specifically periodic oscillations known as harmonic oscillations, plays a crucial role in this context. Mathematically, such motion is described by the equation:

$$x = x_0 \sin (2\pi f \cdot t + \alpha)$$

where  $x_0$  represents the amplitude of the oscillation,  $f$  denotes the frequency, and  $\alpha$  is a constant referred to as the phase, which characterizes the position of the particle at a given instant in time.

The frequency  $f$  corresponds to the number of complete oscillations made by a particle within the medium per unit of time. It is linked to the period of oscillation  $T$  (the period being the time required for one complete oscillation) through the following relationship:

$$f = 1/T.$$

The unit of frequency is known as the hertz  $1 \text{ s}^{-1} = 1 \text{ Hz}$ , (1 *herc*)..

When two distinct vibrations come together, they can undergo a phenomenon called superposition. This means that vibrations with varying frequencies, amplitudes, phases, or even those that are mutually perpendicular, can overlap or combine in some way.

In this discussion, we will explore the superposition of two vibrations that share the same amplitude but have slightly different frequencies, denoted  $f_1$  and  $f_2$ , ( $f_1 > f_2$ ). The arithmetic mean of these frequencies is calculated as follows:

$$f = (f_1 + f_2)/2.$$

If we introduce the notation:  $2\Delta f = f_1 - f_2$ , then we can write the component frequencies:

$$f_1 = f + \Delta f \quad \text{oraz} \quad f_2 = f - \Delta f.$$

The excursions of the individual vibrations are then described by the following formulae:

$$x_1 = x_0 \cos 2\pi f_1 t = x_0 \cos 2\pi(f + \Delta f)t, \quad x_2 = x_0 \cos 2\pi f_2 t = x_0 \cos 2\pi(f - \Delta f)t.$$

The excursion of the resultant vibration is the sum of the excursions of the component vibrations,

$$x = x_1 + x_2 = x_0 \cos 2\pi(f + \Delta f)t + x_0 \cos 2\pi(f - \Delta f)t.$$

After applying the formulae for trigonometric functions of sum and difference of angles, we obtain the expression

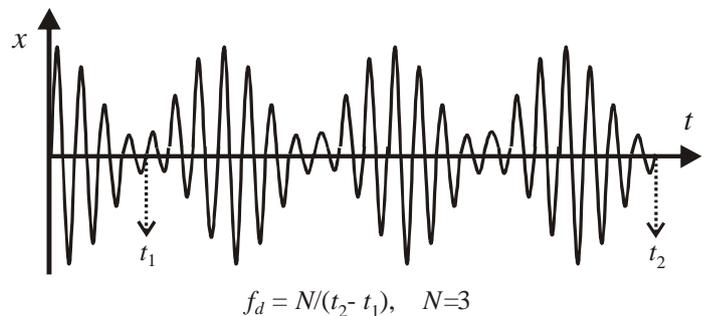
$$x = 2x_0 \cos[2\pi \Delta f t] \cos[2\pi f t],$$

in which the function

$$A = 2x_0 \cos[2\pi \Delta f t]$$

describes the amplitude of the resultant excursion. The frequency of occurrence of the maximum amplitude value ) is the beats frequency. This frequency is twice as high as  $\Delta f$  because the beats maximum occurs for both  $\cos[2\pi \Delta f t]=1$ , and  $\cos[2\pi \Delta f t]=-1$ . The beats frequency is  $f_d = 2\Delta f = f_1 - f_2$ , which is consistent with the statement given at the beginning.

The figure opposite graphically illustrates the beats phenomenon.



**Figure 1) The diagram shows the beginning and end of a cycle of three beats.**

### ***Performance of the exercise***

A dual-channel generator is employed to generate two sound waves with slightly varying frequencies. The resulting waveform is captured as an electrical pulse amplitude by a voltage sensor, which subsequently relays the recorded values to a computer. The temporal evolution of this amplitude is visually represented in the oscilloscope window within the SPARKvue program. Within this program, there is also the capability to measure the frequency of overlapping waves. This measurement is facilitated by the Frequency Spectrum window, accessible via the second tab, which records the spectral distribution of the frequencies under examination.

### ***PART I: Computer Preparation***

1. Begin by powering on the table; locate the table's dashboard situated by your left leg when seated in front of the computer. Rotate the red "knob" in the direction of the arrows until it pops up, akin to turning a car key. Proceed to start the computer and log in as '133.'
2. Launch SPARKvue, then navigate to 'Open Saved Experiment' and choose the file labeled '133' located on your desktop.
3. Upon launching, an initial 'Interference-Beats' window will be displayed, which can be transformed into a 'Frequency Spectrum' window by using the arrow key.
4. The 'Interference Beats' window comprises the oscilloscope screen along with its control interface, serving as the space where we will observe the superposition results.
5. The 'Frequency Spectrum' window, on the other hand, displays the frequency spectrum, facilitating the measurement of the component wave frequencies, including the beats component.

### ***PART II: Setting Up the Measurement System***

1. Begin by connecting the acoustic generator to the main power source. On the right side of the generator, configure the SUMMING AMPLIFIER so that both generator outputs are linked to the volume potentiometer (VOLUME). To achieve this, activate the two middle switches while deactivating the ones at the extremes.
2. Attach the wire ends from the voltage sensor to the designated generator output sockets marked as  $8\ \Omega$  OUT and GND.
3. In parallel with the sensor, connect the speaker by inserting the speaker ends into the sensor plug sockets.
4. Disable the modulation control on each generator by setting the MODULATION switches to OFF.
5. Choose a sine wave on generator No. 1, set the frequency multiplier to 100, and adjust the amplitude value to the middle of the range.
6. Configure generator No. 2 to match the settings of generator No. 1.
7. Power up the generator and adjust the desired volume using the VOLUME knob. Note that this knob also determines the recorded signal amplitude value by the software. When you turn it, the recorded signal amplitude value should approach  $\sim 0V$ .

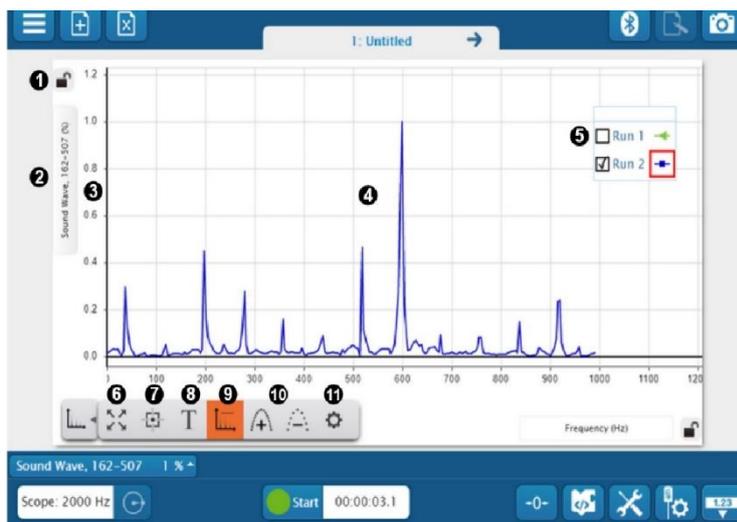
**PART IIIA: SPARKvue graph window**



1. **Axis Lock**  
Click to lock or unlock axis scaling.
2. **Measurement Selector**  
Click to change the displayed measurement or units.
3. **Axis**  
Click and drag to scale each axis individually.
4. **Plot Area**  
Click and drag to move the plot area.
5. **Legend**  
Select a run to analyze by clicking the run in the legend. Right-click the run symbol to change the run color and other data set properties.
6. **Tools Drawer**  
Click to show or hide the Graph tools.
7. **Scale to Fit**  
Automatically scale the axes to fit all data from the selected run within the plot area.
8. **Cursor Mode**  
Click to toggle the function of the cursor.
  - Switch to **Move Mode** to use the cursor to move the plot area.
  - Switch to **Select Mode** to highlight an area on the plot for data analysis.
9. SPARKvue provides two types of coordinates tools:
  - **Coordinates Tool**  
Use to determine the coordinates of a single data point. Use the delta tool to determine the difference between two points of data.
  - **Multi-Coordinates Tool**  
Use to compare coordinates across multiple runs or measurements.
10. **Slope Tool**  
The slope tool provides the slope of a line tangent to a curve at a single point.
11. **Statistics**
  11. **Statistics**  
Use the statistics tool to find the minimum, maximum, mean, and standard deviation of a range of data.
  12. SPARKvue provides two curve fit tools:
    - **Linear Fit**  
Use the linear fit tool to fit a line to data and determine the slope and y-intercept of the trend line.
    - **Curve Fit Tool**  
Use the curve fit tool to fit a curve to data and obtain values about the trend line parameters.
  13. **Annotation Tool**  
Use the annotation tool to label a data point with a brief note.
  14. Use these tools to display multiple y-axes:
    - **Add y-axis**  
Display multiple measurements on the same plot area by adding additional y-axes to a line graph.
    - **Align Origins**  
When displaying more than one y-axis, select to align the origins and scales of each axis.
    - **Add Plot Area**  
Display an additional measurement in a line graph on separate plot area located below the current plot area.
  15. **Prediction Tool**  
Before collecting data, draw a prediction of the results directly on the graph.
  16. **Properties**  
Change the visibility of data points, symbols, and lines connecting data points in a line graph.

**Figure 2) Overview and Description of the SPARKvue graph window (Description Sourced from: <https://help.pasco.com/sparkvue>).**

**PART IIIB: SPARKvue FFT window**



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| <p>1. <b>Axis Lock</b><br/>Click to lock  or unlock  axis scaling.</p> <p>2. <b>Measurement Selector</b><br/>Click to change the displayed measurement or units.</p> <p>3. <b>Axis</b><br/>Click and drag to scale each axis individually.</p> <p>4. <b>Plot Area</b><br/>Click and drag to move the plot area.</p> <p>5. <b>Legend</b><br/>Select a run to analyze by clicking the run in the legend. Right-click the run symbol to change the run color and other data set properties.</p> <p>6. <b>Scale to Fit Tool</b><br/>Change the scale of a line graph to zoom in on a range of data or to view an entire run of data.</p> <p>7. <b>Coordinates Tool</b><br/>Use to determine the coordinates of a single data point. Use the delta tool to determine the difference between two points of data.</p> | <p>7. <b>Coordinates Tool</b><br/>Use to determine the coordinates of a single data point. Use the delta tool to determine the difference between two points of data.</p> <p>8. <b>Annotation Tool</b><br/>Use the annotation tool to label a data point in a graph with a brief note.</p> <p>9. <b>Normalize Data</b><br/>Select to change the vertical scale to make 1 the maximum value.</p> <p>10. <b>Change Sample Rate</b><br/>Click these tools to increase  or decrease  the sample rate.</p> <div style="border: 1px solid #ccc; padding: 5px; margin: 10px 0;"> <p><b>Tip</b><br/>Set a sample rate that is approximately twice the frequency range displayed on the FFT.</p> </div> <p>11. <b>Properties</b><br/>Change the visibility of data points, symbols, and lines connecting data points in a line graph.</p> |
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**Figure 3) Overview and Description of the SPARKvue fft window (Description Sourced from: <https://help.pasco.com/sparkvue>).**

**PART IV: Measuring Procedures**

1. Begin by pressing the Start button to initiate the data measurement process. You should observe a waveform appearing in the Interference - Beats window.
2. **Navigate to the tab labeled 'Frequency Spectrum' (located to the right of the window title). Next, click on the 'Change Sample Rate' icons, as shown in Figure 3, labeled as number 10. Click the '+' icon once, and then click the '-' icon once. Afterward, switch to the 'Interference - Beats' window.**
3. Initially, configure the generators to operate at 100 Hz. Subsequently, adjust the generator frequency to ensure that at least three distinct beats are visible on the graph.
4. Once the measurement conditions are established, press the Stop button to halt the measurement process.

5. Direct your attention below the graph, where you'll find an icon labeled "Tools drawer - Fig.2 number 6." From this toolbox, select the "Add coordinate tool - Fig.2 number 9." You can view tool descriptions by hovering your mouse pointer over them. To determine the frequency of the beats, follow these steps:
  - a. First, position the tool at the beginning of the initial complete cycle of the amplitude change and record the corresponding time displayed by the tool.
  - b. Then, click on the same tool once more and locate the endpoint of the final complete cycle, noting its corresponding time (refer to Fig.1).
6. Calculate and record the number of complete cycles that occur between these two identified points.
7. Switch to the "Frequency spectrum" window by clicking the arrow located to the right of the window caption. This window provides readings of the component wave frequencies obtained from the recorded spectrum.
8. Utilize the "Add coordinate tool" to assist in obtaining readings. Point to the two highest peaks in the spectrum, which correspond to the two component frequencies of the beats. Record these frequencies.
9. Adjust the generator settings to 200 Hz and subsequently to 300 Hz. Repeat the measurement process as outlined in steps 2-6 for each frequency setting.

### ***Data Analysis:***

1. Determine the beats frequency,  $f_d = \frac{N}{t_2 - t_1}$

denoted as N by the number of complete cycles of amplitude change between time points  $t_1$  and  $t_2$ .

2. Compute the difference between the two component frequencies obtained from direct measurements and compare this result with the beats frequency.
3. Calculate the percentage variation between the theoretical and experimental values, represented as  $B_p = \frac{|f_t - f_d|}{f_t} \cdot 100\%$
4. Investigate the potential factors that may have contributed to the disparities between the theoretical and experimental outcomes.