First name.. Date...

Last name.. Degree program name.......................................

Ćwiczenie 119

Exercise 119: Simple Harmonic Motion – Mass on a Spring

Table I: Determining the Spring Constant

Equilibrium position $\boldsymbol{x_0} =$...................... m

Weight	[N]						
x	[m]						
$\Delta x = x - x_0$	[m]						

Spring constant: k = ... N/m.

Table II. Determining the period of oscillation

Total mass added to the string m = ... kg

Peak #		0	1	2	3	4	5	6	7	8
Time	[s]									
Period	[s]	X								

Average period of oscillation determined from table II: $\bar{T} = \sum_{i=1}^{N} \frac{T_i}{N} =$ ________ s

Standard error of the mean for period of oscillation: $S_{\bar{T}} = \sqrt{\frac{\sum_{i=1}^{N}(\bar{T}-T_i)^2}{N(N-1)}}$ =_________s

Percentage difference of oscillation periods' values: $\frac{S_{\bar{T}}}{\bar{T}} * 100\% =$ _________%

Period of oscillation calculated from the formula $T_t = 2\pi\sqrt{\frac{m}{k}}$

T_t, [s]	$B_p = \frac{\lvert T_t - \bar{T} \rvert}{T_t} \cdot 100\%$

Exercise 119: Simple Harmonic Motion – Mass on a Spring

PURPOSE

The objective of this exercise is to explore the motion of a mass undergoing oscillation on a spring.

THEORY

The elastic constant

Suppose we suspend a spring vertically, with its unloaded length denoted as 'l'. Upon applying a mass 'm' to the spring, it undergoes an elongation of 'Δl'. The equilibrium position of the mass now resides at a distance of '$l + \Delta l$' from the spring's suspension point. According to Hooke's law, this elongation 'Δl' is directly proportional to the gravitational force acting on the mass, which is 'mg' (where 'g' represents the gravitational acceleration):

$$mg = k\Delta l \qquad (1)$$

In this equation, the product '$k\Delta l$' is referred to as the elastic force. The constant 'k' is termed the elastic constant and is associated with the spring's resistance to deformation. It serves as a unique characteristic quantity for each spring. The unit of measurement for the elastic constant is *'N/m'* (Newton per meter).

The harmonic motion

Let's now explore the scenario in which a mass, suspended by a spring, experiences a slight downward displacement, denoted as x, from its equilibrium position. In response, the spring exerts an unbalanced elastic force, represented as:

$$\vec{F} = -k\vec{x} \qquad (2)$$

Here, the negative sign signifies that this force opposes the direction of the mass's displacement. Consequently, the elastic force induces oscillations in the mass, causing it to move up and down. In accordance with Newton's second law of motion, which states that an unbalanced force leads to acceleration (F=maF=ma), we can express equation (2) as:

$$ma = -kx \qquad (3)$$

Recalling that acceleration is the second derivative of displacement with respect to time $a = \frac{d^2x}{dt^2}$, we can derive the equation of motion for a mass (mm) oscillating on a spring with an elastic constant 'k' as:

$$m\frac{d^2x}{dt^2} + kx = 0 \qquad (4)$$

The solution to this equation of motion is a harmonic function, given by:

$$x(t) = Asin(\omega t + \varphi) \qquad (5)$$

Here, 'A' represents the amplitude of the motion, signifying the initial deviation of the mass from its equilibrium position. ωω stands for the circular frequency of oscillation, determined by:

$$\omega = \sqrt{\frac{k}{m}} \qquad (6)$$

Lastly, 'ϕ' denotes the initial phase of the motion. As we initiate the motion by displacing the mass to amplitude 'A' at $t=0$ seconds, we must satisfy the condition:

$$x(t=0) = A\sin(\varphi) = A \rightarrow \varphi = \frac{\pi}{2}\,rad = 90^o$$

Consequently, equation (5) takes on the following form:

$$x(t) = Asin\left(\sqrt{\frac{k}{m}}t + \frac{\pi}{2}\right) \qquad (7)$$

Recalling the relationship $\omega = \frac{2\pi}{T} = 2\pi f$ (where 'f' represents the frequency), we can express the period of harmonic motion as:

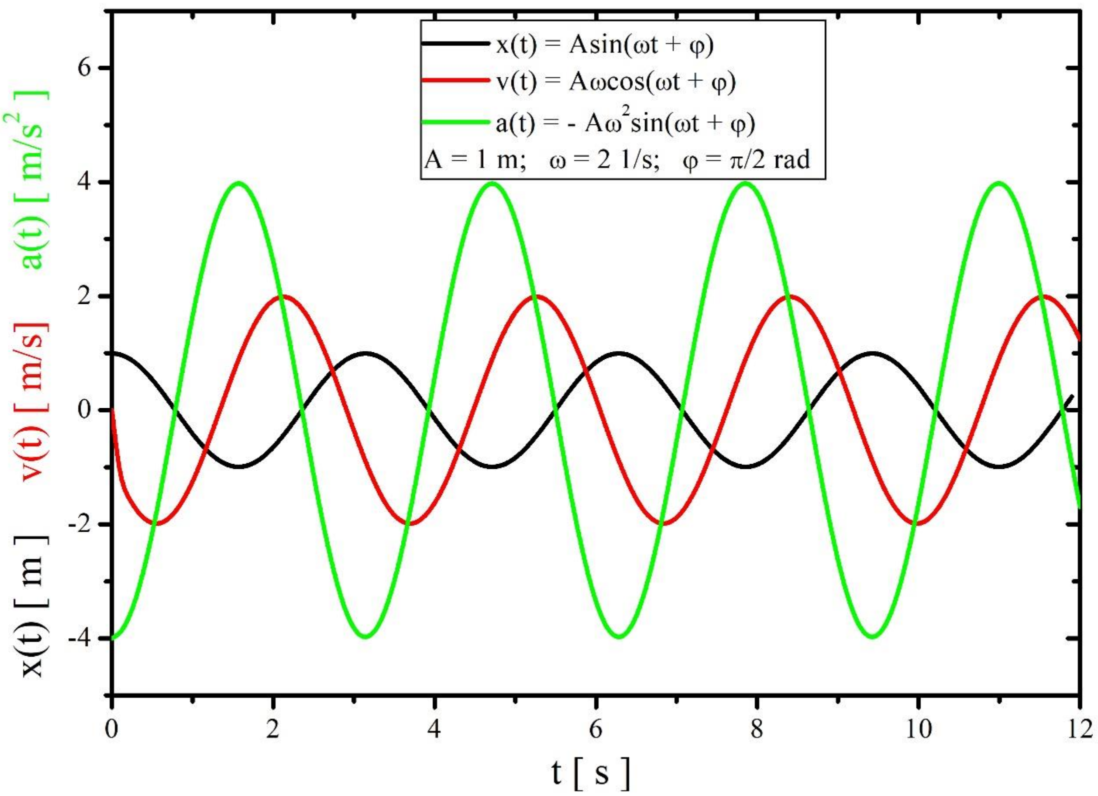
$$T = 2\pi\sqrt{\frac{m}{k}} \qquad (8)$$

By definition, velocity represents the first derivative of displacement with respect to time, while acceleration denotes the first derivative of velocity with respect to time. Therefore, for harmonic motion, we derive the following equations:

$$v(t) = A\sqrt{\frac{k}{m}}\cos\left(\sqrt{\frac{k}{m}}t + \frac{\pi}{2}\right) \qquad (9)$$

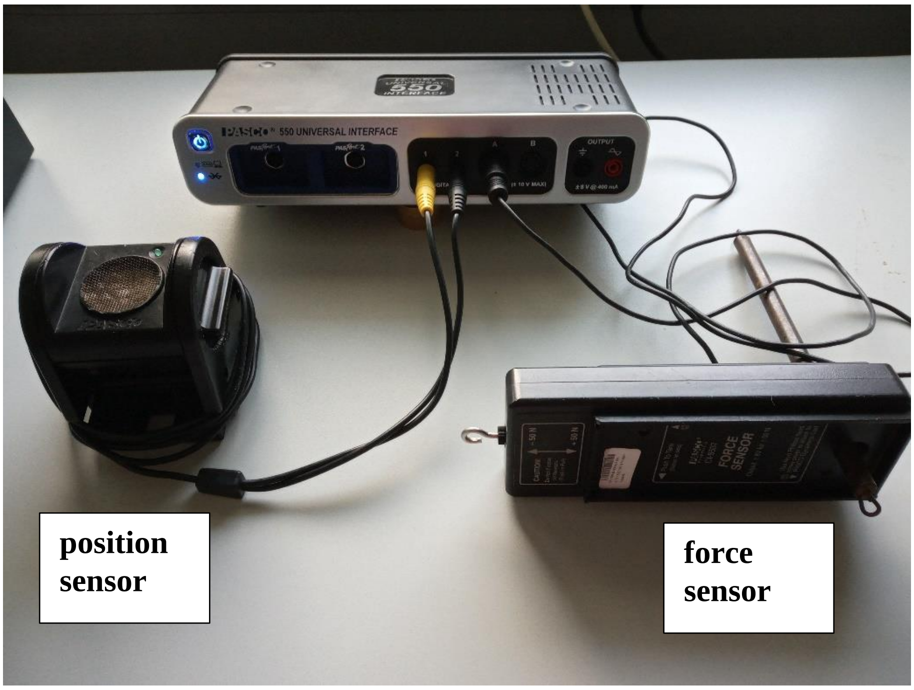
$$a(t) = -A\frac{k}{m}\sin\left(\sqrt{\frac{k}{m}}t + \frac{\pi}{2}\right) \qquad (10)$$

The graph below vividly illustrates the temporal behavior of these position, velocity, and acceleration functions in harmonic motion.

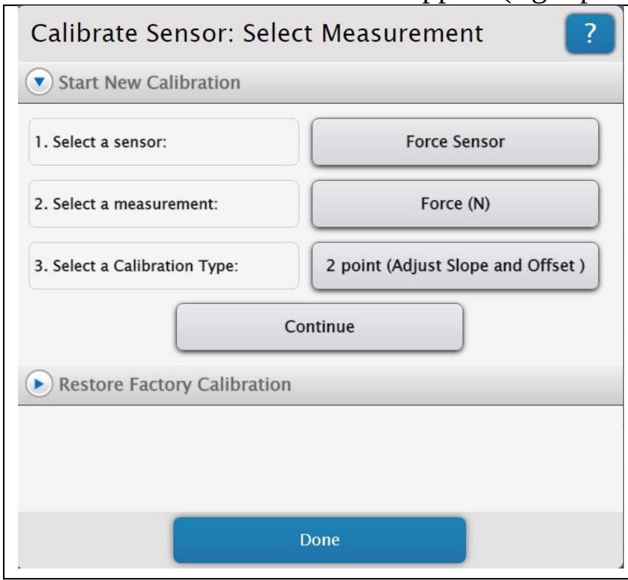


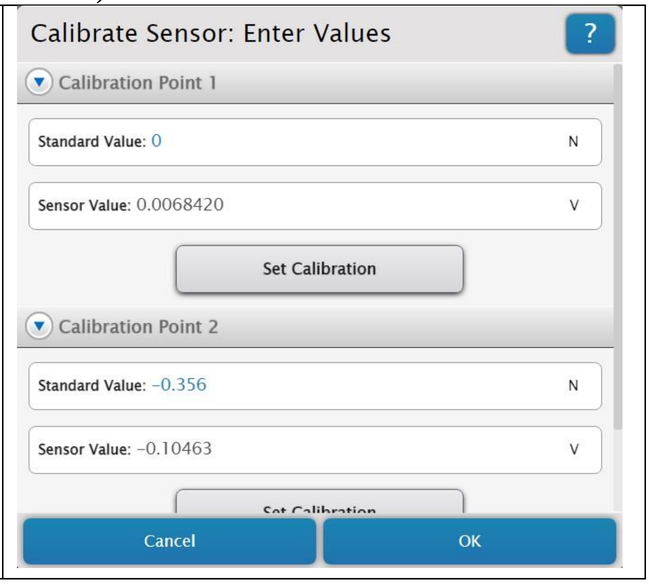
Performing the experiment

1. Turn on the power of the table (see the dashboard of the table - by your right leg, when you sit in front of the computer) - turn the red "knob" in the direction of the arrows (it should pop out), turn the key as in a car and let go.
2. Turn on in the following order: (1) PASCO universal interface, then (2) computer.
3. Connect (if necessary) the force sensor to the analog channel A and the position sensor to the digital channels (yellow tip - channel 1, black tip - channel 2).

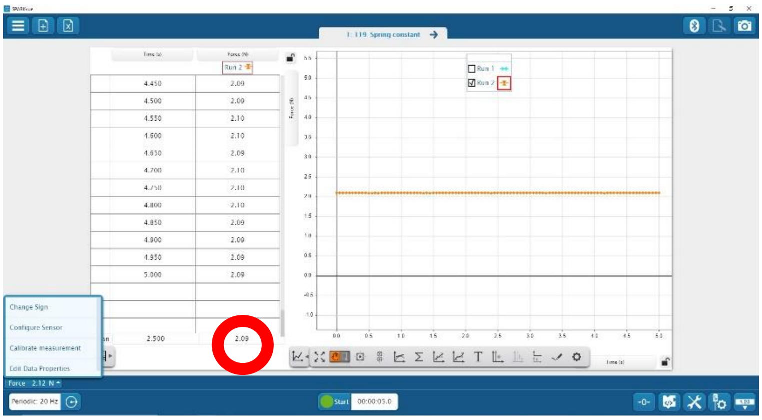


4. Attach the force sensor to the very top of the tripod (if it is in a different location), and locate the position sensor directly under the force sensor on the base of the tripod.
5. Launch SPARKvue, then navigate to 'Open Saved Experiment' and choose the file labeled '**Ex119-SpringConstant**' located on your desktop (see attached photo below for application appearance). Left-click on the icon from the force sensor button (red frame in the photo below) and select the option: "Calibrate measurement" in order to calibrate the force sensor.
6. In the newly opened "Calibrate sensor: Select Measurement" window, select the two-point calibration method (as in the left photo below) and then click "Continue". The "Calibrate sensor: Enter Values" window will appear (right photo below).



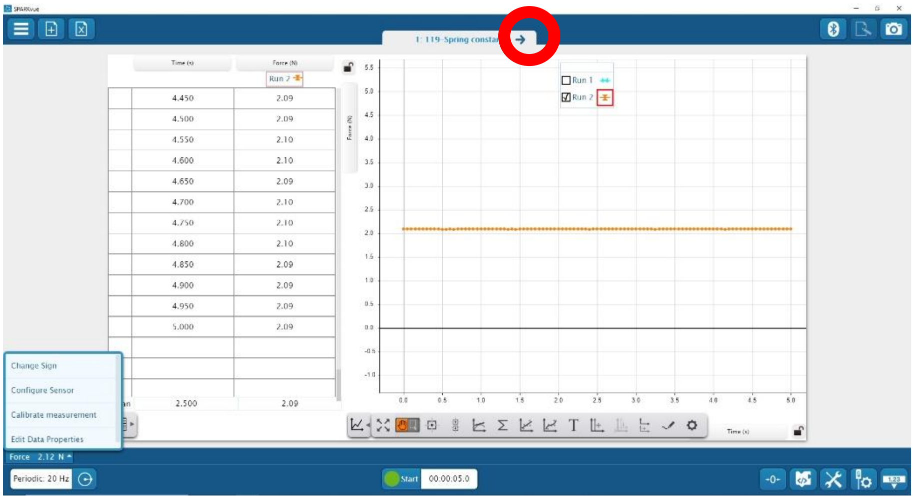


7. For the first calibration point, we enter 0 in "Calibration Point 1" under "Standard Value" and click enter. Then we find the tare reset button on the force sensor, which we press. After this operation, we click "Set Calibration." The program will display the current readings of the force sensor.
8. To determine the second calibration point, take a 0.5 kg weight (it should be at the workstation for exercise 114). We weigh it and multiply the obtained mass by the acceleration of the Earth g=9.81 N/kg. We enter the calculated weight with a minus sign in the "Standard Value" position for the second calibration point "Calibration Point 2". **We should remember that when inputting a number, we use '.' instead of ','.**We hang the weight on the force sensor! After the vibration stops, we click "Set Calibration" for the second calibration point. The program will read the sensor readings and determine the so-called calibration straight line, the coefficients of which can be viewed in the last item of the "Calibrate Sensor: Enter Values" window. **Put back the 0.5 kg weight to the place from which it was taken.**
9. Hang the spring on the hook together with the weight hanger.
10. On the millimeter scale read the position of the end of the spring (the last coil). Record the result of measuring the equilibrium position of the spring x_0 in the table.
11. **Tar the indication of the force sensor - the button TARE on the side of the sensor**.
12. Suspend a mass of 10 g (weigh the masses available in the exercise beforehand) and read the new position of the end of the spring (the last coil) and record it in the table in the row marked by x.
13. Read the weight force of the suspended mass - click the Start button in the application. The program will read the force sensor readings and make average of them. Enter the value indicated in the photo below into the table. The reading will be correct despite the small vibration of the weight.

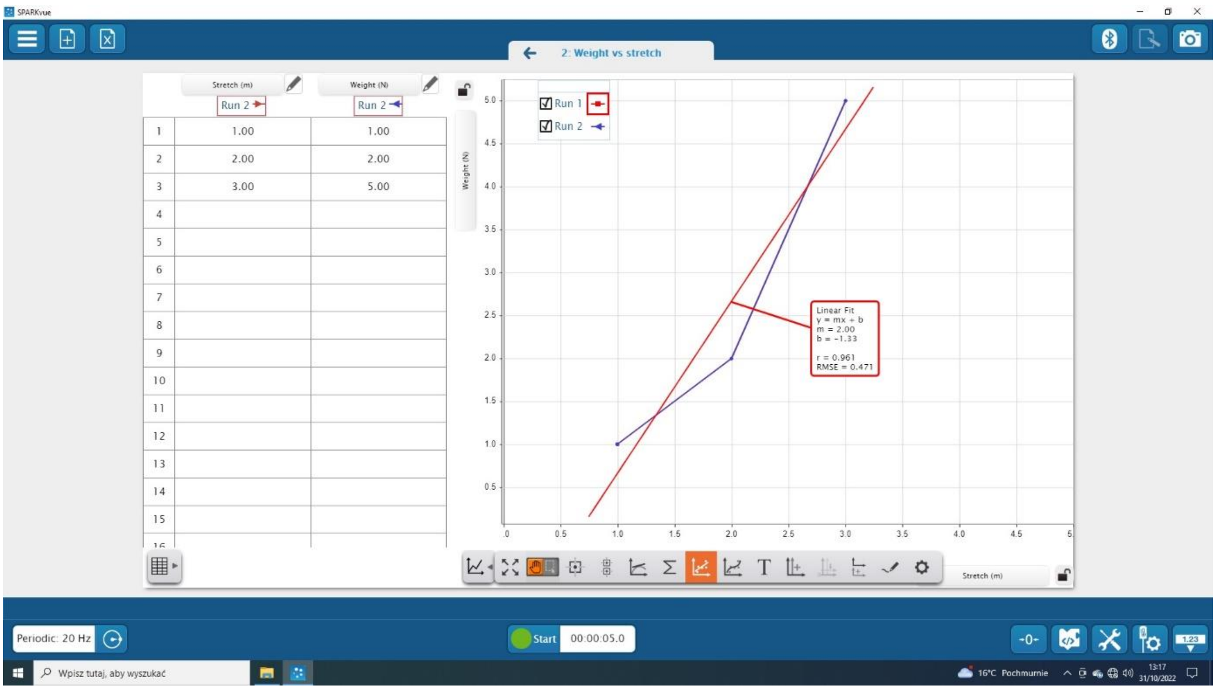


14. Increasing the mass, in 10 g increments, enter in the table the new position of the end of the spring and the new weight measured with the force sensor. **We should remember that when inputting a number, we use '.' instead of ','.**
15. After completing the table, calculate the spring deflection calculated by subtracting the extension of the unloaded spring:$\Delta x = x - x_0$

16. Next, change the tab in the application by clicking the arrow as indicated in the following photo.

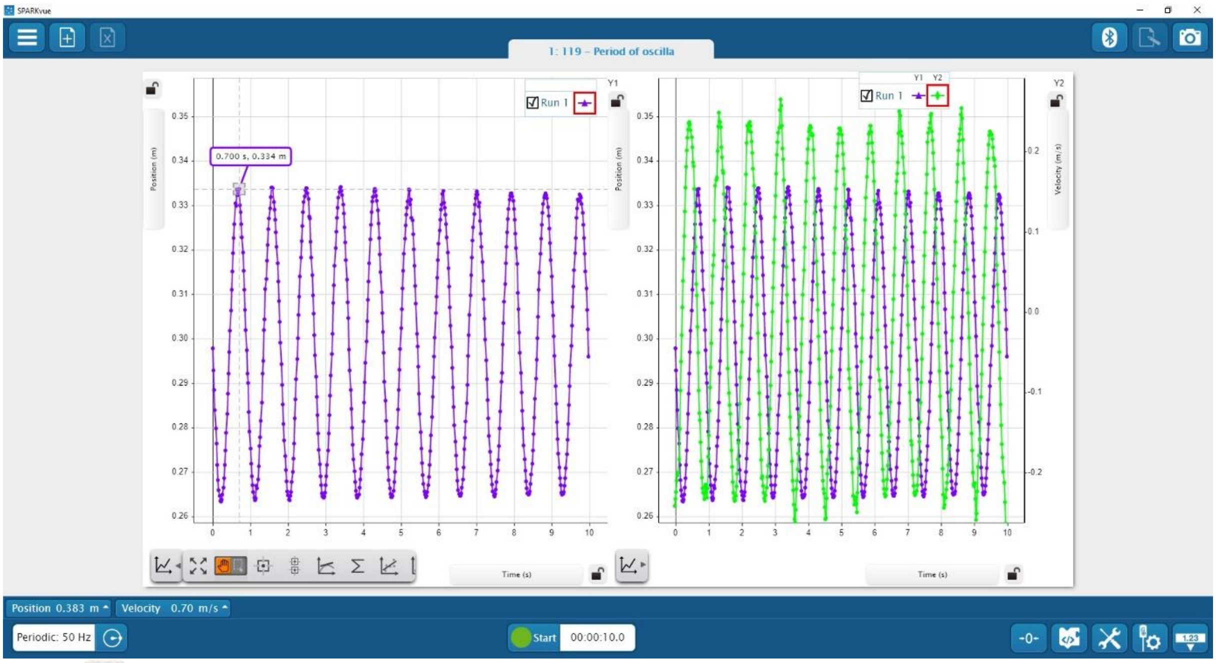


17. Enter the measured deflections and weights in the application table, and then fit the straight line to the resulting graph by clicking the appropriate icon in the application (photo below). **We should remember that when inputting a number, we use '.' instead of ','.** The determined directional coefficient of the straight line is, in this case, the spring constant - we record it in the table.



18. Close the application.
19. Launch SPARKvue, then navigate to 'Open Saved Experiment' and choose the file labeled ' **Ex119-Period of oscilation** ' located on your desktop.
20. Do not change the setup! I.e., we do not pull off the force sensor, and the position sensor should be on the stand of the tripod directly under the hanging weights.
21. Put a mass, in the range of 30-60 g, on the spring.
22. Stretch the spring with the mass from the equilibrium position by about 2 cm.

23. Start the recording by clicking the Start button. The program will measure the deflection in time (left and right graphs), as well as the change in velocity of the oscillating mass (right graph).



24. Enter  - tool to read the coordinates of a point from the graph. We set this tool to show us the amplitude. We enter the first coordinate into the table, which is the time for nine consecutive peaks.
25. Find the period of each oscillation (calculate the difference between the times measured for consecutive peaks). Calculate the average period of the oscillation T. Record the result obtained.
26. Calculate the standard error of the mean for this measurement according to the formula:

$$S_{\bar{T}} = \sqrt{\frac{\sum_{i=1}^{N}(\bar{T} - T_i)^2}{N(N-1)}}$$

27. Calculate the percentage error for the measurement using the formula:

$$\frac{S_{\bar{T}}}{\bar{T}} * 100\%.$$

28. Calculate the theoretical value of the period of oscillation $T_t = 2\pi\sqrt{\frac{m}{k}}$, using the measured value of the spring constant and the value of the mass (in kg) suspended on the spring. Find the percentage difference between the calculated and measured value of the period: $B_p = \frac{|T_t - \bar{T}|}{T_t} \cdot 100\%$
29. Review the questions for "discussion", to include in the ‘conlusions’ part of the report.
30. Close the application and turn off the computer.

Questions for discussion

1. How does your calculated value for the period of oscillation compare to the measured value for the period of oscillation? Consider the percent difference between your calculated value and the measured value.
2. When the position of the mass is farthest from the equilibrium point, what is the velocity of the mass? Check it in the Position and Velocity graph from **Ex119-Period of oscilation.**
3. When the absolute value of the velocity of the mass is greatest, where is the mass relative to the equilibrium position? Check it in the Position and velocity graph from **Ex119-Period of oscilation.**