First name .... Date ....

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Exercise 413

Determination of liquid's viscosity coefficient using Stoke's method

The solid spheres method

Liquid: GLYCERINE	Radius of the cylinder R = ... m	
Liquid's density, ρ_f	[kg/m^3]	
Mass of n = ... spheres, m	[kg]	
Average mass of 1 sphere, m_s	[kg]	
Volume of n spheres, V	[m^3]	
Volume of 1 sphere $V_s = \frac{V}{n}$	[m^3]	
Average radius of 1 sphere, r	[m]	

Distance	$s_1 =$ .... m			$s_2 =$ .... m		
Time, [s]						
Average time	$t_1 =$ .... s			$t_2 =$ .... s		
Velocity [m/s]	$u_1 =$			$u_2 =$		
Viscosity coefficient [Pa·s]	$\eta_1 =$			$\eta_2 =$		
Average viscosity coefficient, [Pa·s]				$\eta =$		

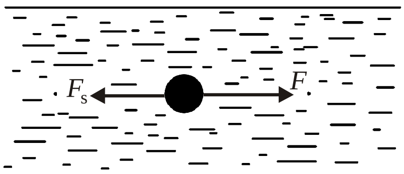
Theoretical value

of viscosity coefficient ………………..… [Pa·s] for the temperature of ……….. °C

Exercise 413. Determination of liquid's viscosity coefficient using Stoke's method

Introduction

Imagine immersing a sphere in a liquid and starting to pull it at a constant velocity. Let's consider what the resistance of the liquid to the moving sphere depends on, that is, when you have to pull harder and when you have to pull weaker to maintain the given velocity. If we switch to a larger sphere, the resistance increases. Therefore, the drag force depends on the radius of the sphere. If we increase the velocity of the ball, the resistance also increases. The drag force also depends on the type of liquid, specifically on its viscosity. The more viscous the liquid, the greater the drag force.

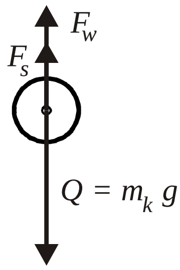


To be precise, the drag force, acting on a rigid sphere moving in an unconstrained viscous fluid in a slow uniform motion, is defined by Stokes' law. It states that the drag force F_s is directly proportional to the velocity v of the ball, its radius r and the viscosity coefficient of the fluid η, and that the proportionality coefficient (in the case of a sphere) is equal to 6π:

$$F_s = 6\pi r u \eta \quad (1)$$

Stokes' law can be used to determine the viscosity coefficient. If a ball of radius r, and velocity v encounters resistance F_s, then from equation (1) the value of η can be calculated.

Let us now consider the descent of a sphere in a liquid. A falling sphere in a liquid is subject to three forces: gravity $Q = mg$, viscous drag F_s and buoyancy F_w. Initially, the force of gravity is greater than the sum of the other forces and the sphere falls in an accelerated motion with increasing velocity u. But as the velocity increases, according to Stokes' law, the viscous drag increases more and more and at a certain point the force of gravity becomes equal to the sum of $F_s + F_w$. From this point onwards, the further descent of the sphere continues in a uniform motion. Now let us describe the equilibrium condition for the forces causing the uniform motion of the sphere:



$$m_s g = F_s + F_w \quad . \quad (2)$$

According to *Archimedes' law*, the buoyancy force is equal to the weight of the liquid displaced by the body immersed in it. If the volume of the sphere is V_s, and the density of the liquid is ρ_f, then the buoyancy force is equal to

$$F_w = V_s \rho_f g \quad (3)$$

We substitute F_s with equation (1) and F_w with equation (3) in the force equilibrium condition (2),

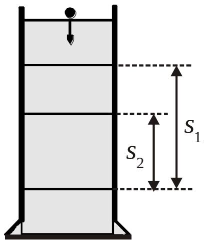
$$m_k g = 6\pi r u \eta + V_s \rho_f g,$$

and after transformation we obtain the formula for the viscosity coefficient:

$$\eta = \frac{(m_s - V_s \rho_f) g}{6\pi r u}. \quad (4)$$

Equation (4) is only valid for expanding liquids, i.e. those contained in very wide vessels. If a sphere falls in a cylindrical tube of radius R, the effect of the vessel's surface reduces the fall velocity and a correction factor, depending on the ratio $\frac{r}{R}$, must be introduced into equation (4). The corrected formula for determining the viscosity coefficient is as follows:

$$\eta = \frac{(m_s - V_s \rho_f) g}{6\pi r u \left(1 + 2{,}4 \frac{r}{R}\right)}. \quad (5)$$



Performance of the task

The exercise involves determining the viscosity coefficient of glycerine. In the experiment we will use a glass cylinder filled with the liquid being investigated. We will use glass spheres to test our glycerine.

We determine the following quantities:

- **the radius of the cylinder $\boldsymbol{R}$**, by measuring its internal diameter with a ruler,
- **the density of liquids $\boldsymbol{\rho_f}$**, by using an hydrometer appropriate for the liquid (the density of oil is less than that of water ρ_w, and the density of glycerine is greater than ρ_w),
- **the volume of a glass sphere $\boldsymbol{V_s}$**, measured by the change in water level after dropping a minimum of 10 glass spheres into a measuring cylinder (10 ml capacity) filled with water to approximately $\frac{2}{3}$ of its height,
- **the weight of $\boldsymbol{n}$ spheres $\boldsymbol{m}$**, by weighing a minimum of 10 glass spheres,
- **the radius of a sphere $\boldsymbol{r}$,** by using the formula for the volume of a sphere $r = \sqrt[3]{\frac{3V_s}{4\pi}}$,
- **the velocity $\boldsymbol{u}$**, we mark two different distances $s_1$and s_2 on the cylinder and measure the falling times of the spheres with a stopwatch. The spheres are dropped through a funnel. Measure several times for each distance. Calculate the average falling time t_1 (for distance s_1) and t_2 (for distance s_2).

$$u_i = s_i / t_i \,, \quad i = 1, 2 \,.$$

Calculation of the uncertainties

We calculate the relative errors in the determination of the viscosity coefficient using the method of the exact differential which we apply to equation (5). We assume that the quantities subject to measurement error are: u, m_s, V_s, ρ_s, r, while we ignore the error in measuring the radius R of the cylinder due to its negligible effect on the final value of the $\Delta\eta$.

After calculating the partial derivatives and performing the appropriate transformations, we get:

$$\boldsymbol{\frac{\Delta\eta}{\eta} = \frac{\Delta u}{u} + \frac{\Delta m_s + \rho_f \cdot \Delta V_s + V_s \cdot \Delta\rho_f}{m_s - \rho_f V_s} + \frac{1 + 4,8(\frac{r}{R})}{1 + 2,4(\frac{r}{R})} \cdot \frac{\Delta r}{r}}$$

Measurement errors of physical quantities are calculated (for measurement $i = 1$ or $i = 2$) as follows:

- $\frac{\Delta u_i}{u_i} = \frac{\Delta s_i}{s_i} + \frac{\Delta t_i}{t_i}$, where Δs_i – accuracy of the distance measurement, $\Delta t_i = \sqrt{\frac{\sum_{k=1}^{n}(t_i - t_{is})^2}{n(n-1)}}$ (standard error of the mean time of descent of the sphere for n measurements taken for one of the two distances),
- Δm_s - weighing accuracy divided by the number of spheres weighed,
- ΔV_s is defined as twice the accuracy of the volume of liquid in the measuring cylinder divided by the number of spheres,
- $\Delta\rho_s$ is equal to the smallest graduation on the scale of the hydrometer (1 kg/m^3),
- $\frac{\Delta r}{r} = \frac{\Delta V_s}{3V_s}$ - the logarithmic derivative method was applied to the formula $r = \sqrt[3]{\frac{3V_s}{4\pi}}$.