First name .... Date ....

Last name .... Degree program name ....

Exeriment 409

Measurement of Young's modulus with a thickness gauge

The speed value is set in the thickness gauge v' = ……………………. m/s

Material						
The shape of the cross-section						
Poisson's ratio μ						
Dimensions	*a,b*	[m]				
	d					
Sample length	*l*	[m]				
Sample mass	*m*	[kg]				
Volume	*V*	[m^3]				
Density	*ρ*	[kg/m^3]				
Readout of the thickness gauge measurement	*l′*	[m]				
Actual velocity	*v*	[m/s]				
Young's modulus	*E*	[GPa]				
Young's modulus - a tabular value	E_{tab}	[GPa]				
Absolute error related to tabular value	ΔE_{tab}	[GPa]				
Absolute error	ΔE	[GPa]				

* see *Tips at the end of this manual*

Experiment 409. Measurement of Young's modulus with a thickness gauge

Definition of Young's modulus

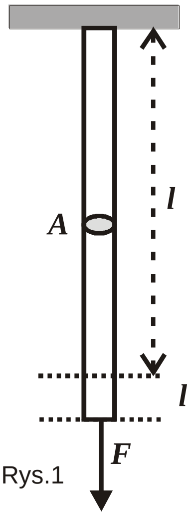
If we apply force to an immobilized elastic body, *stresse* σ will arise in this body, causing its deformation. The stress σ in a bar with a cross-section A, on which a force $\vec{F}$ acts (perpendicular or tangent to *A*) is equal to the ratio of the force to the cross-sectional area of the bar:

$$\sigma = F/A \tag{1}$$

Stress resists the intermolecular forces inside the material. There are usually three types of stress: stretching (lengthening the body), compressive (shortening the body) and shear (deforming the shape of the body). In the latter case, the force acts tangent to the cross-sectional area.

The change in rod length *l* due to tension or compression is proportional to its length. If, for example, a bar of length l, stretched by force $\vec{F}$, increases its length by *l*, Fig. 1, then the measure of deformation Δ*l*, is the relative change in length:

$$\varepsilon = \Delta l / l \,. \tag{2}$$



Rys.1

When the body returns to its dimensions after removing the force $\vec{F}$, this deformation is called elastic. For small deformations, ε is proportional to σ:

$$\varepsilon = \frac{1}{E} \cdot \sigma \,, \tag{3}$$

where *E* is the *modulus of elasticity* (called *Young's modulus*) of a material. The linear relationship between stress and strain is known as *Hooke's law*. After substituting into (3) the formulas defining ε and σ, we get:

$$\Delta l = \frac{1}{E} \cdot \frac{l}{A} F \,. \tag{4}$$

Thus, Hooke's law states that when stretching or compressing, the change in length is proportional to the force acting.

Young's modulus is expressed, like stress or pressure, in pascals: 1 Pa = 1 N/m^2.

Measurement n of Young's modulus with ultrasound

Sound waves are mechanical vibrations of the medium's molecules propagating in a medium. The source of the sound is areas of the medium in which, for some reason, mechanical vibrations, i.e. fluctuations in stress or pressure, occur. There can be different types of sound waves in solids. In gases and liquids, the vibrations of molecules associated with the sound wave only take place in the direction of the wave's motion (it is therefore the longitudinal wave).

According to the range of frequencies received by the human ear, vibrations of the medium's particles are divided into: *infrasound* (0÷16 Hz), sounds (16 Hz ÷20 kHz) and *ultrasound* (from 20 kHz). The physical laws concerning the above-mentioned types of vibrations are the same, but their properties (e.g. interaction with living matter) are different.

There are two known methods of producing ultrasound. One of them uses the *magnetostriction*: rods made of ferromagnetic bodies (e.g. iron, nickel) placed in an alternating magnetic field experience changes in their length in time together with changes of the magnetic field. The vibrations are particularly strong when the natural frequency of the bar coincides with the frequency of the field changes. The rod then performs resonant vibrations and becomes a source of ultrasound. In this way, ultrasound with a frequency of up to 60 kHz can be produced. The second method of producing ultrasound is based on the use of the *reverse piezoelectric phenomenon.*

Certain crystals, e.g. quartz, placed in an electric field whose direction coincides with the corresponding crystal axis, change their geometrical dimensions in line with changes in the electric field. The size of the crystal is selected so that it performs resonant vibrations, i.e. that the frequency of its natural vibrations is consistent with the frequency of changes in the electric field.

A thickness gauge is an instrument that measures the time taken by ultrasound to travel through a sample to and, after reflection, back to t_0, divides it into half $t=t_0/2$, multiplies it by the set velocity value v ', programmed for the selected material, and displays the resulting sample thickness value l'

$$l' = v' \cdot t.$$

In this experiment, we want to calculate the **unknown** velocity of sound in the material and then the Young's modulus, so we perform the inverse operation:

- for the tested material, on the basis of the assumed velocity v' and the thickness l' **measured by the thickness gauge**, we calculate the ultrasound transit time through the sample t,

$$t = \frac{l'}{v'}$$

- on the basis of the **real** sample length l, (measured with calipers), and the time t calculated above, we determine the speed of sound v

$$v = \frac{l}{t}$$

In short: the unknown real speed of sound in the material is calculated from the proportion:

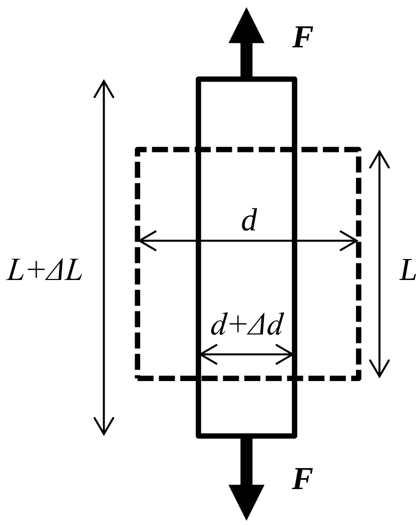
$$v = v' \cdot \frac{l}{l'}$$

The velocity v of the sound wave in a piece of material with a size large in comparison with the wavelength, is given by the formula:

$$v = \sqrt{\frac{E(1-\mu)}{\rho(1+\mu)(1-2\mu)}}$$

where:

E - Young's modulus, ρ - the density of the tested material,
μ - **Poisson's ratio** of the tested material.



The Poisson's ratio is a dimensionless physical quantity that describes the transverse strain ratio for longitudinal deformation under axial stress.
If a longitudinal bar with a diameter d of length L is stretched (or compressed) by a length ΔL (see figure) under the force F, then its diameter will change by the amount Δd, given by the formula:

$$\Delta d = -\mu d \frac{\Delta L}{L}$$

This effect should be taken into account for bars with a diameter greater than the wavelength λ propagating in a given element. In the case of a thickness gauge using ultrasonic frequency 5 MHz, wavelength λ is of the order of 0.5 mm (depending on the material). The rods used in this experiment are much wider than the wavelength λ.

Performance of the task

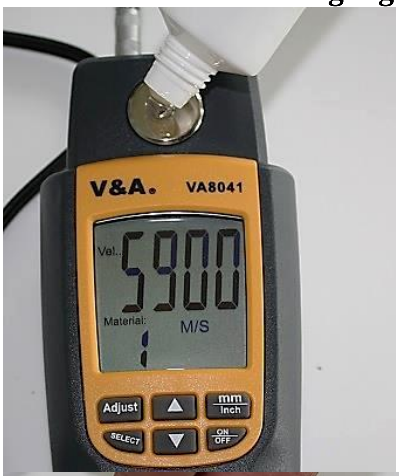
1. The thickness gauge consists of a main panel with a digital display and a transmitting / receiving head.

   In the experimental set, the head is properly connected with wires – please, do not disconnect plugs.

   The panel with buttons looks like in the picture on the right. We only use buttons [ON/OFF], [SELECT] and [Adjust], and a calibration disc**.**

   Please do not use the other buttons, in the photo they are crossed out

2. Turn on the thickness gauge by clicking
3. On the button ON/OFF.
4. The assumed speed value v' in meters per second will be displayed on the screen. Can change it with the button [SELECT]. **Choose the material with the highest possible speed** $v' > 5000$ m/s.
5. Before the first measurement, we **calibrate the thickness gauge** - see photos



Please, apply a small (one drop) amount of coupling paste to the calibration disc.

It is an activity analogous to that performed by a doctor during an ultrasound examination - the paste / liquid ensures the ultrasound transmission from and to the transducer.

6. Apply the heads tightly to the disc and hold.

7. Press and hold the button [`Adjust`]. There will be four minus signs on the screen and they will gradually disappear: `----`, `---`, `--`, as the calibration progresses.

8. If the device has been properly calibrated, we will see the image on the screen as shown in the picture on the right - the indicated disc thickness will be 4.0 mm, or in the range 3.9-4.1 mm.

9. Make the **proper measurement of the sample**

- the photo shows a measurement for a copper cylinder.

10. The object is placed **on the table on the sponge layer** provided in the set to avoid ultrasounds being transferred to the table top.

11. Grease the end of the rod with coupling paste and **lightly** press the head

12. After the result is displayed - the mark is visible , we write down the measured length value *l'*.

13. Please, measure the real length l of the sample with calipers.

15. Measure the transverse dimensions of the sample and its mass (on scales with a range of up to 1 kg).
16. Repeat the measurements (without calibrating the meter) for the remaining materials.

Please, save the results of measurements read from measuring instruments (calipers, scales and thickness gauges) with full accuracy displayed by the instruments.

Please, write down the calculation results with 4 significant figures

Calculations

The actual speed of sound v is calculated on the basis of the assumed speed of sound v', programmed in the thickness gauge, of the length indicated by the thickness gauge l', and the real length measured with calipers *l*, from the formula:

$$v = v' \cdot \frac{l}{l'}$$

The density of the tested material is calculated from its mass *M*, measured with laboratory scales, and the volume *V*, calculated from the sample size

$$\rho = \frac{M}{V}$$

The volume V is determined from the formula:

$$V = l \cdot S$$

where l –the length of the sample is measured with a calipers, S –the cross-sectional area of the sample calculated from the formula for the area of the appropriate geometric figure.

The Young's modulus for the tested substance is calculated on the basis of the material density ρ, the speed of sound in the material v, and the Poisson's ratio from the formula:

$$E = v^2\rho \ \frac{(1+\mu)(1-2\mu)}{(1-\mu)}$$

The quantities ρ, v have been calculated earlier.

The Poisson's ratio for the various materials is given in the table attached to the sample set.

Calculation of the uncertainties

The uncertainty of determining the Young's modulus ΔE can be calculated from the formula:

$$\frac{\Delta E}{E} = 2\frac{\Delta v}{v} + \frac{\Delta \rho}{\rho}$$

The uncertainty of the measured speed of sound is calculated from the formula:

$$\frac{\Delta v}{v} = \frac{\Delta l}{l} + \frac{\Delta l'}{l'}$$

accept Δl —based on the accuracy of calipers
(electronic calipers 0,02 mm, mechanical calipers 0,1 mm).

The maximum error of the thickness gauge indications is assumed as:

$$\Delta l'/l' = 1\%.$$

Assuming that the weighing accuracy is high, the density uncertainty is calculated as:

$$\frac{\Delta\rho}{\rho} = \frac{\Delta V}{V}$$

Accuracy of determining the volume of the examined bodies ${}^{\Delta V}/_{V}$ is equal 2 %

DISCUSSION

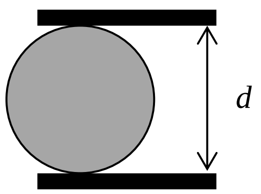
1. Compare the obtained results with the data from tables of physical quantities. Can the difference between the obtained results and the tables results be explained by the accuracy of the measurements, i.e. whether $\Delta E_{tab} < \Delta E$? ($\Delta E_{tab} = |E_{tab} - E|$)

2. What is the difference between the results obtained in this experiment and the tables values caused by?

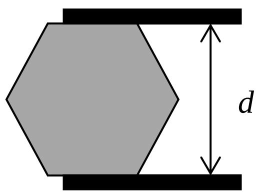
<u>Tips</u>

Measurement method and calculation of the cross-sectional area S for round, hexagonal, triangular and rectangular bars (black rectangles = calipers jaws):

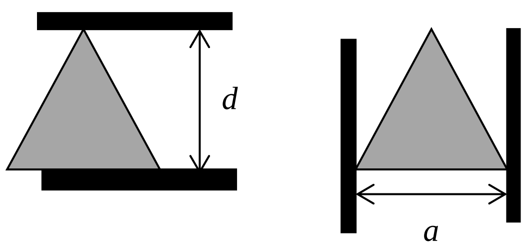
Circle: $S = \frac{1}{4}\pi d^2$



Hexagon: $S = \frac{\sqrt{3}}{2}d^2$



Triangle: $S = \frac{1}{2}da$



Rectangle: $S = ab$