Department of Physics and Biophysics; Institute of Biology

First name .... Date ....

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Exercise 408

Determination of the moment of inertia of the body using the physical pendulum

Type of metal solid	m, [kg]	Internal diameter D_i, [m]	$l = \frac{D}{2}$, [m]	t_i, [s]	$T = \frac{t}{n}$, [s]	I, [kg×m²]	I_s, [kg×m²]
Hoop	1,453						
Wheel – axis I	1,404						
Wheel – axis II	1,404						

The number of measured, complete oscillations of the solid: n = ...

<u>Designations in the table</u>:

D – mean diameter value, l – the distance of the center of gravity from the suspension point, t_i – time of n complete oscillations of the solid, t – mean time of n oscillations, T – period of oscillation, I and I_s –moments of inertia of the body with respect to the suspension point and its centre of gravity.

Exercise 408. Determination of the moment of inertia of the body using the physical pendulum

Introduction

To describe the rotational motion of rigid bodies, the concept of the moment of inertia was introduced, which in this case is a measure of inertia, similar to mass in translational motion. This is because in the rotational motion of a solid it is not only the mass that matters, but also its distribution with respect to the axis of rotation. The rotational motion of the whole solid can be considered as the sum of the circular motions of the individual elementary masses into which the entire solid can be divided

The moment of inertia of an elementary mass ￼ is expressed by the formula

$$I_i = m_i \cdot r_i^2,$$

r_i - the distance of the elementary mass m_i from the axis of rotation, Fig.1.: *The total mass moment of inertia* of the body is the sum of the moments of inertia of the elementary masses:

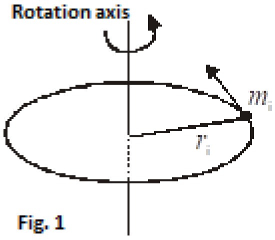

Fig. 1

$$I = \sum_{i=1}^{\infty} m_i r_i^2$$

If a solid moving in the rotational motion is subjected to a moment of force $\vec{M}$, the solid moves with angular acceleration $\vec{\varepsilon}$, the value of which is proportional to the value of the moment of force and inversely proportional to the moment of inertia of the body.

$$\varepsilon = M/I \; . \tag{1}$$

Equation (1) expresses the content of the second law of dynamics for rotational motion.

Determining the moment of inertia of the body with respect to selected axis of rotation involves measuring the period of oscillations of physical pendulum. A *physical pendulum* is a rigid body suspended on a horizontal axis passing through the point O, located above the body`s center of mass S, Fig. 2. The *period of oscillation of a pendulum* is the time it takes for the body to return to its original position after one complete swing.

If the body is deviated from the equilibrium position by a small angle α, it will start to move with an oscillating motion with period *T*. The pendulum motion can be considered as a special case of rotational motion with variable angular acceleration ε. Therefore, to describe the motion we may use the principle of dynamics of rotational motion expressed by equation (1).

Fig. 2 shows that the value of the moment of force acting on the solid deviated from the equilibrium position by the angle α, isα

$$M = -mg \cdot l \sin \alpha \tag{2}$$

The "-" sign comes from the fact that the moment of force acts opposite to the direction of the solid's deflection from the equilibrium position.

We substitute the relation (2) to (1):

$$\varepsilon = -\frac{mgl}{I} \sin \alpha . \tag{3}$$

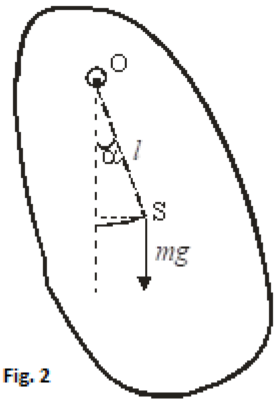

Fig. 2

Taking into account in equation (3) that for small angles α (in radians) there is an approximate equality $\sin \alpha \approx \alpha$ and by substituting *e* by the second derivative of a with respect to time (definition of angular acceleration) we obtain:

$$\frac{d^2\alpha}{dt^2} = -\frac{mgl}{I} \alpha . \tag{4}$$

The relation (4) is the equation of harmonic motion (acceleration is proportional to the coordinate *a* and has the opposite sign with respect to it). The solution of this equation is the function

$$\alpha = \alpha_0 \cos(\omega t + \varphi)\ , \tag{5}$$

where α_0 is the amplitude of the oscillation (the largest deviation from the equilibrium position) φ – the initial phase of the motion (the phase determines the value of the angle α at time t = 0), and ω it is the circular frequency of the oscillation.

As can be easily checked from the condition $\alpha(t) = \alpha(t+T)$, ω satisfies the relation

$$\omega = 2\pi/T = 2\pi f\ , \tag{6}$$

f –frequency of oscillation (the reciprocal of the oscillation period *T*, i.e. the number of complete oscillations in 1s).

By differentiating equation (5) twice we get:

$$\frac{d^2\alpha}{dt^2} = -\omega^2\alpha. \tag{7}$$

Comparing the sides (4) and (7), we get the condition that ω must satisfy:

$$\omega^2 = \frac{mgl}{I}. \tag{8}$$

If we substitute $\omega = 2\pi/T$ in equation (8), we obtain the formula for the moment of inertia of the body:

$$I = \frac{mgl \cdot T^2}{4\pi^2}. \tag{9}$$

Determination of the moment of inertia therefore comes down to the measurement of the period of oscillation of the solid.

The moment of inertia I_s with respect to the axis passing through the center of mass of the solid is of great importance in the rotational motion. I_s cannot be measured by using the physical pendulum (the moment of gravity is zero). It can be calculated using the parallel axis theorem, also known as Huygens–Steiner theorem, or just as *Steiner's theorem* which states:

The moment of inertia of the body with respect to any axis, *I*, is equal to the moment of inertia with respect to the parallel axis and passing through the center of gravity of the body I_s, increased by the product of the mass of the solid multiplied by the square of the distance between this axes, $I = I_s + ml^2$. That is

$$I_s = I - ml^2. \tag{10}$$

Performance of the task

Perform measurements for the hoop and metal wheel.

1. Measure the inner diameter D_i .of the solid three times in different sections with a calliper. Calculate the average value of the solid`s radius - *l*.
2. Hang the block on a tripod, tilt it slightly from the equilibrium position and, after the sliding stops, measure the time of few tens of complete oscillations. Repeat the measurement three times and calculate the average oscillation period *T*.
3. Calculate the moment of inertia *I*, formula (9), and I_s, formula (10).

Calculation of the uncertainties

The absolute error ΔI is calculated using the logarithmic derivative method based on the formula (9):

$$\frac{\Delta I}{I}=\frac{\Delta m}{m}+\frac{\Delta g}{g}+\frac{\Delta l}{l}+\frac{2\Delta T}{T}\,. \tag{11}$$

Assume $\Delta m = 0$ (we do not weigh the solids) and $\Delta g = 0$. Formula (11) simplifies to the form

$$\frac{\Delta I}{I}=\frac{\Delta l}{l}+\frac{2\Delta T}{T} \tag{12}$$

Using the formula (12) calculate the absolute error ΔI and the relative percentage error

$$B_p=\left(\Delta I/I\right)\cdot 100\%$$

Calculate the values Δl and ΔT: $\quad \Delta l=\dfrac{\max\left|D-D_i\right|+0{,}1\mathrm{mm}}{2}, \quad \Delta T=\dfrac{\max\left|t-t_i\right|}{n}; \quad \mathrm{i}=1{,}2{,}3\ .$