First name .... Date ....

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Exercise 407 Study of vibrational motion of the elastic pendulum

I. Determination of the spring constant

Block		Location of the indicator		Elongation	Coefficient
mass m_i, [kg]	weight Q_i, [N]	without load l_{0i}, [m]	with load l_i, [m]	$x_{0i} = l_i - l_{0i}$, [m]	$k_i = Q_i / x_{0i}$, [N/m]
The average value of the spring constant, $k = (k_1 + k_2 + k_3)/3$					

II. Checking the isochronism law of the pendulum

Amplitude A_i [m]	Time of $n =$ .... vibrations, t_i, [s]			Period of vibration $T = \bar{t}/n$, [s]

III. Determination of the mass of the block

Time of $n =$ .... vibrations, t_i	[s]			
Period of a vibration $T_i = t_i / n$	[s]			
The average period of vibrations, T	[s]			

Mass of the spring, m_s	[kg]		The mass of the indicator, m_w	[kg]	
Calculated mass of the block, m_x	[kg]		Measured mass of the block	[kg]	
Absolute error	[kg]		Relative error	[%]	

Exercise 407. Study of vibrational motion of the elastic pendulum

Introduction

Simple harmonic motion

One type of motion that is often found in physics is an *oscillating motion* in which the body moves back and forth along the same path. For example, a pendulum in a clock, a weight suspended on a spring, or atoms in a crystal lattice move with vibrating movements. A special example of an oscillating motion is simple harmonic motion.

Simple harmonic motion occurs when the force acting on a vibrating body is proportional to the body's deflection from the equilibrium position and directed towards it. The equation of motion of a point with mass *m* subjected to such a force is as follows:

$$ma = -k\,x\,, \tag{1}$$

where *a* is the acceleration of mass *m*, *x* –its inclination from the equilibrium position, k – coefficient of proportionality. If we take into account that the acceleration *a* can be expressed as the second derivative of the *x* coordinate at time *t*, the equation (1) is as follows:

$$a = \frac{d^2x}{dt^2} = -\frac{k}{m}x\,, \tag{2}$$

whose solution is a function:

$$x = A\cos(\omega t + \varphi)\,. \tag{3}$$

Therefore, we can say that a simple harmonic motion is a motion in which the coordinate describing the motion of the body changes periodically in a sinusoidal manner. The quantities occurring in the above function characterize the harmonic motion:

φ –*the initial phase* of movement; it is the angle that determines the *x*-coordinate value of the moment $t = 0$.

A –vibration amplitude is the maximum deflection of the vibrating body from the equilibrium position,

ω –the *circular frequency of vibrations* satisfying the relationship

$$\omega = 2\pi/T = 2\pi f\,, \tag{4}$$

where *T* means a period, and *f* – a frequency of vibration

The period of vibration is the time during which one complete vibration is made. During the period, the body goes through each point of its path twice (except for the extreme points) and returns to its initial state. The *frequency of vibrations* is the number of vibrations per unit of time. The period and frequency are related:

$$f \cdot T = 1.$$

We get the speed and acceleration in harmonic motion by calculating the derivatives:

$$v = \frac{dx}{dt} = -A\omega\sin(\omega t + \varphi), \qquad a = \frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t + \varphi)\,.$$

The last expression, after substituting (3), can be written:

$$a = -\omega^2 x\,. \tag{5}$$

Equation (5) shows the relationship between the coordinate of the body position, its acceleration, and the circular frequency of vibrations in harmonic motion.

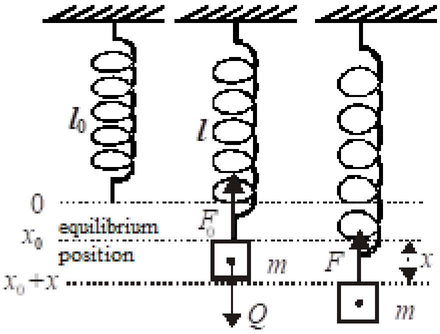
Equating equations (2) and (5), we obtain the relationship

$$\omega = \sqrt{k/m}\,, \tag{6}$$

which shows that the circular frequency is equal to the square root of the quotient of the coefficient *k* and the body mass *m*.

The harmonic motion of a body suspended on a spring

Our experiment aims to observe the harmonic movement of a weight suspended on a spring, the so-called *elastic pendulum*, fig. 1. A block suspended on a spring at rest is in a position which is called the equilibrium position. If the block is pulled down below the equilibrium position and released, it will begin to vibrate up and down.



Rys.1 Wahadło sprężynowe

Two opposing forces act on the block (weight) resting in the equilibrium position and balance each other. They are the force of gravity $\vec{Q} = m\vec{g}$ operating vertically downward and the restoring force $\vec{F}_0$ of the stretched spring directed in the direction opposite to the deformation. According to *Hook's law*, at small deformations, the elastic force is proportional to the deformation: x_0:

$$F_0 = -k\,x_0\,. \tag{7}$$

The constant *k* here means *spring constant*. The coordinate of deformation x_0 is obtained from the condition of the equilibrium of forces: $\vec{Q} + \vec{F}_0 = 0$, from which it follows that $mg = kx_0$. If we measure x_0 for a known mass *m* suspended on a spring, we can determine its spring constant *k*:

$$k = \frac{mg}{x_0}\,. \tag{8}$$

When the block (weight) is tilted *x* up or down from the equilibrium position, an unbalanced spring force *F* appears, proportional to *x* (Hook's law):

$$F = -k\,x\,. \tag{9}$$

As we already know, this type of force is responsible for the harmonic motion of the body - so we can use the compounds (4) and (6), from which we obtain the formula for the period of vibrations:

$$T = 2\pi\sqrt{\frac{m}{k}}\,. \tag{10}$$

We can see that the period of vibrations depends only on the mass of the weight *m* and the constant *k* of the spring, and not on the initial deviation of the weight from the equilibrium position. The fact that the vibration period does not depend on the amplitude *A* is referred to as the *isochronism law of the spring pendulum*. Checking the law of isochronism will be one of the goals of the experiment.

When deriving the formula for the period, the air resistance force was neglected, which causes a decrease in the amplitude of vibrations and influences the period of vibrations. Due to the small amplitude of the vibration velocity, we will still not take it into account in the calculations.

The second simplification is the omission of the mass of the spring m_s. From energetic considerations of vibrating motion, a formula can be derived that takes into account the fact that the spring has its own mass. The "corrected" formula for the pendulum oscillation period is:

$$T = 2\pi\sqrt{\left(m + \frac{1}{3}m_s\right)\Big/k}\,. \tag{11}$$

If we determine the period of vibrations of an unknown mass suspended on a spring with a known *k* value, we can use formula (11) to determine the mass of the weight *m*.

Performance of the task

Determination of the spring constant

1. Hang the spring on a tripod and attach a light plastic indicator to its end. On the linear scale, connected to the tripod, read the position of the horizontal line marked on the indicator.
2. Attach a block of known mass to the indicator and read the position of the indicator line again. Calculate the elongation of the spring.
3. Calculate the weight of the suspended block and the spring constant *k* - formula (8).
4. Perform the measurements for three blocks with different masses and calculate the average value of *k*.

Checking the law of isochronism of the pendulum.

1. Load the spring with a weight, pull it down, e.g. by 1 cm, and measure the time of several dozen (*n*) full vibrations of the pendulum. For the same vibration amplitude, we repeat the measurements three times. Calculate the average time of vibration *n* and then the period of the oscillating motion.
2. Repeat the period measurements at two other, suitably increased vibration amplitudes.

Attention: The spring load must be selected appropriately for checking the isochronism law. If the mass *m* is too small, the vibrations are too fast and we are not able to count them, and if the mass is too high - the vibrations are not harmonic (the elongation does not comply with Hook's law).

Determination of the mass of the block

Similarly, as above, we determine the period of vibration of a spring loaded with an unknown mass m_x. By transforming (11) we get:

$$m = k\frac{T^2}{4\pi^2} - \frac{1}{3}m_s .$$

The mass *m* is the sum of the mass of the block m_x and the mass of the indicator. The mass m_x is calculated from the formula:

$$m_x = m - m_w = k\frac{T^2}{4\pi^2} - \frac{1}{3}m_s - m_w . \qquad (12)$$

Calculation of the uncertainties

The measurement error of the *k* coefficient is calculated as the maximum error of the average of three measurements:

$$\Delta k = \max\left|k - k_i\right|, \quad i = 1, 2, 3.$$

The estimation of the mass measurement error m_x is performed using the total differential method:

$$\Delta m_x = \frac{T^2}{4\pi^2}\Delta k + k\frac{2T}{4\pi^2}\Delta T + \frac{1}{3}\Delta m_s + \Delta m_w .$$

$\Delta m_s = \Delta m_w$ — spring and indicator weighing accuracy

$\Delta T = \frac{\max\left|\bar{t} - t_i\right|}{n}$; (*i* = 1, 2, 3) —error in determining the period