First name .... Date ....

Last name .... Degree program name ....

Exercise 403

Determination of the sound speed in the air using the method of the acoustic resonance

Determining the Wavelength of a sound wave of a frequency $f =$ .... Hz

Measurement No.	0	1	2	3	4	5	6
Position of the piston during resonance L_i, [m]							
Wavelength $\lambda_i = 2 \cdot \Delta L_i$, [m]							
Average wavelength: $\bar{\lambda} = \frac{\lambda_1 + \lambda_2 + ... + \lambda_N}{N}$, [m]							

Attention: A "zero" measurement means a reading during the first resonance.

Calculation of the speed of sound in the air

Air temperature	T	[K]	
The speed of sound in the air	$v = \bar{\lambda} \cdot f$	[m/s]	
Theoretical value of the speed of sound	$v_t = v_0 \sqrt{T/T_0}$	[m/s]	
Relative error (concerning the theoretical value)	$B_p = \frac{\|v - v_t\|}{v_t} \cdot 100\%$	[%]	
Mean square error	σ	[m]	
Relative error from the error calculus	$\frac{\Delta v}{v} \cdot 100\%$	[%]	

Exercise 403: Determination of the sound speed in the air using the method of the acoustic resonance

REQUIRED EQUIPMENT	• Thermometer
• Resonance tube with generator	• Universal meter
	• Two electric wires

Purpose

The purpose of this exercise is to determine the wavelength of the sound. For the measurement, we will use the phenomenon of acoustic resonance in a tube in which a wave of a certain frequency propagates. The data obtained makes it possible to calculate the speed of sound in the air.

Theory

The formation of mechanical waves

If we move any part of an elastic medium from its equilibrium position, it will vibrate around this position. Thanks to the elastic properties of the medium, these vibrations are transferred to the next molecules of the medium, which also begin to vibrate. In this way, the disorder spreads through the entire medium.

Wave motion is the propagation of a disturbance in a medium.

Waves that arise and propagate in elastic media are called *mechanical waves.*

The wave reaching a given point in the medium causes it to vibrate, transmitting energy to it, which is supplied by the source of vibrations. The energy of the wave is the kinetic energy and the potential energy of vibrations of the medium particles. With the help of waves, energy can be transferred over long distances. Transferring energy without transferring substances, or mass, is called *energy transport*.

Depending on the direction of vibration of the medium particles in relation to the direction of wave propagation, *longitudinal* and *transverse waves* are distinguished.

A wave is *transverse* when the direction of vibration of the medium particles is perpendicular to the direction of wave propagation and the direction of energy transport. An example may be the vibrations of a taud cord, which we cyclically move up and down the end of a cord.

A wave is *longitudinal* when the direction of vibrations of the medium particles is parallel to the direction of wave propagation and the direction of energy transport. An example of this are sound waves in the air.

The wave is characterized by the following basic quantities:

- ✓ *Wave amplitude A* is equal to the absolute value of the maximum displacement or distance moved by a point on a vibrating body or wave measured from its equilibrium position.
- ✓ *Wave phase* φ determines the value of the displacement at x in time t
- ✓ *The period of vibrations* T of the medium molecules through which the wave passes is the time in which the particle will make one complete vibration
- ✓ *Wave frequency f* is the number of complete vibrations per unit time.

The period and frequency are related: $f = 1/T$. The unit of frequency is hertz (Hz). At a frequency of 1 Hz, the particle makes 1 vibration in 1 s.

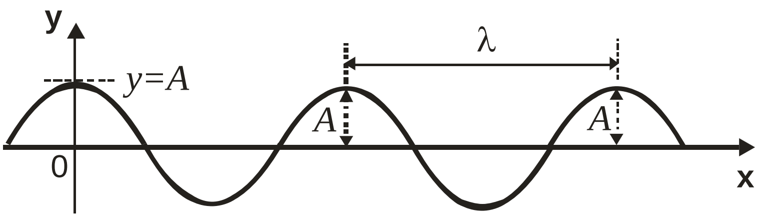
The path that the wave travels during one period of vibrations is called the *wavelength* - λ.

The speed of the wave is therefore equal to:

$$v = \lambda/T = \lambda \cdot f \tag{1}$$

The speed of the wave is equal to the product of the wavelength and the frequency of the vibration.

When the wave travels the path λ, the particle at the starting point will make 1 complete vibration, so it will be in the same phase as the particle that the wave just reached.

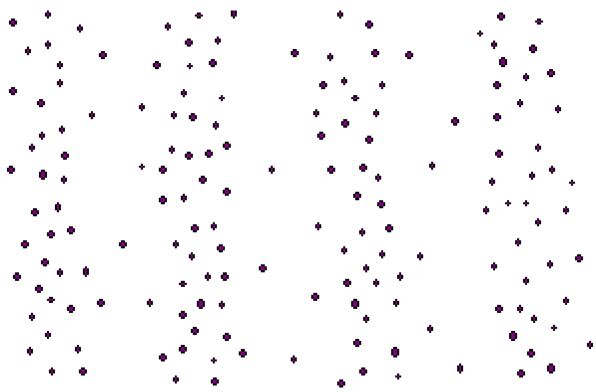


The wavelength of a wave is the distance between two nearest points of equal deflection and direction of movement.

Sound waves

Sound waves are a special type of mechanical waves. A sound wave is any mechanical longitudinal wave. A person perceives the sound with waves at frequencies from 20 Hz to 20,000 Hz. Both the upper and lower limits of the perceived frequencies can be an individual human feature.

A sound wave in air consists of propagating perturbations that cyclically condense and dissolve the air. (cyclic change of pressure and density). These areas of condensation and dilution travel in a certain direction at the speed of sound and may fall into someone's ear and give the impression of sound.



Sounds with a frequency greater than 20 kHz are called ultrasounds. Some animals like dogs (up to 35kHz), bats (up to 100kHz), and dolphins (200kHz) can hear them.

Sounds with frequencies below 20 Hz are called infrasound. The infrasound category includes seismic waves propagating in the interior of the Earth.

Under constant conditions, the speed of sound in various media is relatively stable and defined. The speed of sound depends on the density of the medium, its elastic properties (for solids), compressibility, and temperature (for liquids).

The speed of sound in air depends on temperature (in Kelvin) according to the relationship:

$$v = v_0\sqrt{T/T_0} \qquad (2)$$

where $v_0 = 331{,}5\ \mathrm{m/s}$ —the speed of sound in air at temperature $T_0 = 273{,}15\ \mathrm{K}$, $(0°\mathrm{C})$.

A change in air temperature by 10^{o}C causes a change in the speed of sound by about 6 m/s (the density of the air changes with the temperature).

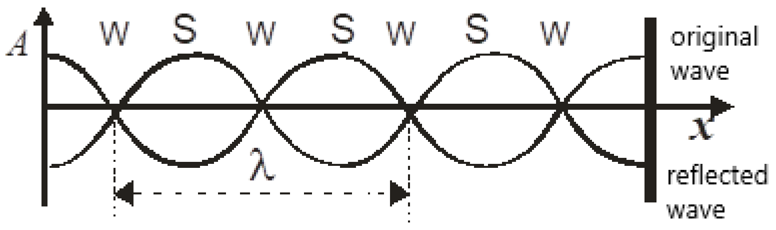
Acoustic resonance

When the loudspeaker diaphragm vibrates near the pipe, the pipe resonates at certain sound frequencies - a standing wave then forms in the tube. A standing wave is created in a tube when the wave reflected from the end of the tube interferes with the wave sent from the source. *Wave interference* is the overlapping of waves of the same frequency, which strengthens or weakens the intensity of the resultant wave.

In fact, the sound wave in the tube is reflected back and forth several times between the ends of the tube. In general, the successively reflected waves are not consistent in phase and the amplitude of the resultant wave will be small. At certain vibration frequencies, all reflected waves are in phase and give a large standing wave amplitude. These frequencies are called *resonant frequencies.* At these frequencies there is the maximum energy transfer between the loudspeaker and the given length of pipe.

A standing sound wave in the air has nodes and antinodes. The **nodes** in a standing sound wave are the points where there is almost no air vibration. The **antinodes** are the points at which the air movement is at its maximum.

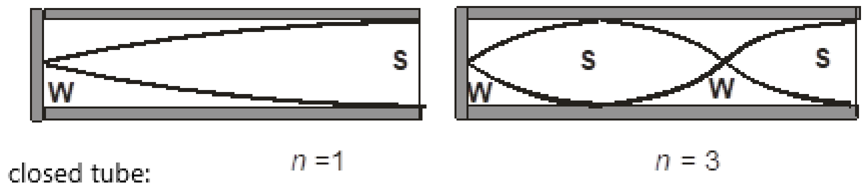
The distance between successive nodes or antinodes is half the wavelength $\lambda/2$.



standing wave as the result of interference of the original wave and reflected wave; W -nod; S- antinod

The reflection of the sound wave can take place at both the open and closed ends of the tube. If the end of the pipe is closed, the air molecules cannot vibrate beyond the plane of the closure - there are nodes at the end of the pipe. When a wave is reflected at the open end, there are antinodes. The conditions for resonance are easier to understand using the term wavelength. We will formulate them for a tube closed at one end.

If the tube is closed on one side (open only on the loudspeaker side), resonance occurs when the length of the tube L is equal to an odd multiple of a quarter of the wavelength λ.



$$L = n\frac{\lambda}{4}; \quad n = 1,\ 3,\ 5,\ 7,...$$

For a tube closed on one side, the resonance condition at wavelength λ is therefore as follows:

$$\lambda = \frac{4L}{n}; \quad n = 1,\ 3,\ 5,\ 7,.... \qquad (3)$$

Using the equations (1) and (3), we can calculate the length of the pipe L at which the sound resonance with a given frequency *f* will occur. For a pipe closed on one side, we get:

$$L = \frac{n\,v}{4\,f}; \quad n = 1,\ 3,\ 5,\ 7,.... \qquad (4)$$

Measurement of the wavelength

The exercise will test the resonance of sound with a constant frequency *f* **in a tube closed at one end**. The pipe is closed with a plunger that can be moved along the pipe. The distance of the face of the piston from the end of the tube at which the loudspeaker is attached is a measure of the actual length L of the resonance tube. When this distance allows a standing wave to form in the pipe, resonance arises and a loud sound comes from the pipe, which means that the length L is exactly equal to an odd multiple of a quarter of the wavelength λ. As the plunger is moved away from the end of the loudspeaker tube, i.e. the distance L is extended, the resonance disappears, and when the tube length increases to half the wavelength, the resonance reappears:

$$\Delta L = \frac{v}{4f} \cdot \Delta n = \frac{\lambda}{4} \cdot 2 = \frac{\lambda}{2}$$

(Δn – a difference of adjacent odd numbers).

Reading two adjacent positions of the plunger (coordinates) at which resonance occurs makes it possible to calculate of the half wavelength value sound propagating in the tube.

After determining the wavelength, we calculate the speed of sound in the air at room temperature from the dependence (1).

Performance of the task

A loudspeaker, connected to an acoustic frequency generator, causes the air to vibrate in the resonance tube. The signal generator can produce several voltage frequency values transmitted to the loudspeaker. The microphone, mounted at the mouth of the resonance tube (next to the loudspeaker), produces a DC voltage corresponding to the sound volume. The signal recorded by the microphone can be read on a voltmeter connected to the microphone.

Measurement activities

1. Move the piston in the resonance tube close to the generator.
2. Connect the cable of the frequency generator into a power outlet 230 V and select one of seven with the frequency dial possible frequency values.
3. Set the multimeter to measure frequency and connect it to the sockets marked on the generator *„Częstotliwość, Hz”(Frequency Hz).*
4. Turn on the generator and read and write down the value of the selected frequency (in the next part of the exercise, do not change the frequency of the generator).
5. Turn off the generator and disconnect the multimeter.
6. Turn the multimeter knob to DC voltage measurement and connect it to the sockets on the generator marked with *„Głośność, V DC” (sound volume, V DC).*
7. Turn the generator back on and **start slowly** pushing the plunger away from the speaker. When the sound volume is at its maximum, precisely adjust the position of the plunger while observing the voltmeter indications. The maximum voltage value represents exactly the resonance position (the maximum voltage value may be slightly different for each resonance).
8. Write the plunger position coordinates in the table (with an accuracy of 1mm).
9. Move the piston away again and set the coordinate corresponding to the next resonant position. We repeat the measurements until we get five to six readings.
10. Turn off the generator and multimeter, and read and write air temperature.

Data analysis

1. Calculate the difference of adjacent ΔLi readings for successive resonances and determine for each wavelength difference: $\lambda_i = 2 \cdot \Delta L_i$.
2. Calculate the average wavelength and the speed of sound in air $v = \bar{\lambda} \cdot f$.

3. Calculate the theoretical speed of sound based on the approximate formula (2) describing the dependence of the speed of sound in air of temperature (temperature in Kelvin!).

4. Calculate the relative error of the determined value concerning the theoretical value v_t:

$$B_p = \frac{|v - v_t|}{v_t} \cdot 100\% .$$

Calculation of the uncertainties

Let us denote successive measurement results by λ_i, where index *i* denotes the measurement number ($i = 1, \ldots, N$). The arithmetic mean of the measurement results is a good estimate of the sound wavelength value:

$$\bar{\lambda} = \frac{1}{N}\sum_{i=1}^{N}\lambda_i = \frac{\lambda_1 + \lambda_2 + \ldots + \lambda_N}{N}$$

As a measure of the measurement uncertainty of the arithmetic mean of the value λ, we take the mean square error σ (the so-called standard deviation of the mean value)::

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}\left(\bar{\lambda} - \lambda_i\right)^2}{N\left(N-1\right)}}$$

We assume that the error of the determined value λ is σ, which can be written as $\lambda = \bar{\lambda} \pm \sigma$.

A relative error in determining the speed of sound in air, calculated on the basis of the formula $v = \lambda \cdot f$, is equal to the sum of the relative error $\Delta\lambda/\lambda = \sigma/\bar{\lambda}$ and the relative error of the frequency measurement f with a digital meter:

$$\frac{\Delta v}{v} = \frac{\Delta\lambda}{\lambda} + \frac{\Delta f}{f} = \frac{\sigma}{\bar{\lambda}} + \frac{\Delta f}{f} .$$

In the case of digital meters used in the exercise, the accuracy (relative error) of the frequency measurement should be equal to $0{,}3\,\%$.

The result of the calculation of the measurement error should be rounded up.

Finally, in conclusions, the relative error of the speed of sound obtained from the error calculus should be compared with the error relative to the theoretical value.