First name
Date
Last name $\qquad$

## Experiment 402

## Determination of the buoyant force and density of objects

| PHYSICAL QUANTITIES | UNITS | CYLINDER (enter sample number) | CUBOID (enter sample name) |
| :---: | :---: | :---: | :---: |
| $V_{\mathrm{p}}$Initial volume | [ $\mathrm{cm}^{3}$ ] |  |  |
|  | [m ${ }^{3}$ ] |  |  |
| Final volume | [ $\mathrm{cm}^{3}$ ] |  |  |
|  | [m ${ }^{3}$ ] |  |  |
| $V=V_{\mathrm{k}}-V_{\mathrm{p}}$ <br> Object volume | $\left[\mathrm{m}^{3}\right]$ |  |  |
| Object mass | [g] |  |  |
|  | [kg] |  |  |
| $F_{\mathrm{m}}=\mathrm{m} \cdot \mathrm{~g}$ <br> Object weight | [N] |  |  |
| Submerged object mass | [g] |  |  |
|  | [kg] |  |  |
| $F_{M}=M \cdot g$ Submerged object weight | [N] |  |  |
| $F_{\mathrm{w}}=F_{\mathrm{m}}-F_{\mathrm{M}}$ <br> Buoyant force | [ N ] |  |  |
| $\rho_{\mathrm{m}}=m / V$ <br> Object density | $\left[\mathrm{kg} / \mathrm{m}^{3}\right]$ |  |  |
| $\rho_{\mathrm{H} 2 \mathrm{O}}=F_{\mathrm{w}} /(V \cdot g)$ <br> Water density calculated from buoyant force | $\left[\mathrm{kg} / \mathrm{m}^{3}\right]$ |  |  |
| $\begin{gathered} F_{\mathrm{T}}=V \cdot \rho_{\mathrm{Tw}} \cdot g \\ \text { Theoretical buoyant force } \end{gathered}$ | [ N ] |  |  |

## Experiment 402. Determination of the buoyant force and density of objects

## PURPOSE OF THE EXPERIMENT

The purpose of the experiment is determination of the buoyant force and density of objects using their mass and the gravity force in the air and liquid.

## THEORETICAL INTRODUCTION

Objects with the same volumes but made of dissimilar materials, have different masses and weights.

Mass ( $\mathbf{m}$ ) is a quantitative measure of matter in a physical object. It is a constant value that does not change even when the object is placed on another planet. In the International System of Units (SI), the base unit of mass is the kilogram [kg] and the mass measurements are obtained with a balance.

The weight (Force of gravity, $F_{m}$ ) of an object is the force acting on the object due to gravity.

$$
\begin{equation*}
F_{m}=m \cdot g \tag{1}
\end{equation*}
$$

where: $F_{\mathrm{m}}=$ force of gravity (object weight) [ N ]
$m=$ object mass [kg]
$\mathrm{g}=$ standard gravity $(\mathrm{g}=9.81)\left[\frac{\mathrm{m}}{\mathrm{s}^{\mathrm{z}}}\right]$
The unit of measurement for weight is that of force, which in SI is the Newton [N]. At constant gravitational acceleration, the weight is proportional to the mass of object.

Density (volumetric mass density or specific mass, $\rho$ ) is the substance's mass (m) per unit of volume (V).

$$
\begin{equation*}
\rho=\frac{m}{V} \tag{2}
\end{equation*}
$$

where: $\rho=$ density $\left[\frac{\mathrm{kg}}{\mathrm{m}^{\mathbf{s}}}\right]$
$m=$ mass [kg]
$V=$ volume $\left[\mathrm{m}^{3}\right]$
The specific weight (unit weight, $\gamma$ ) is the weight per unit volume of an object

$$
\begin{equation*}
\gamma=\frac{m \cdot g}{V} \tag{3}
\end{equation*}
$$

where: $\gamma=$ specific weight $\left[\frac{N}{m^{5}}\right]$
$m=$ mass [kg]
$g=$ standard gravity $\left[\frac{\mathrm{s}^{2}}{}\right]$
$V=$ volume [ $\mathrm{m}^{3}$ ]
The specific weight depends on the value of the gravitational acceleration, which varies with location.

The specific weight ( $\gamma$ ) of an object is defined as the product of its density $(\rho)$ and the standard gravity (g)

$$
\begin{equation*}
\gamma=\rho \cdot g \tag{4}
\end{equation*}
$$

where: $\gamma=$ specific weight $\left[\frac{N}{m^{\mathrm{s}}}\right]$

$$
\begin{aligned}
& \rho=\text { density }\left[\frac{\mathrm{kg}}{\mathrm{k}^{\mathrm{s}}}\right] \\
& g=\text { standard gravity }\left[\frac{m}{g^{2}}\right]
\end{aligned}
$$

When the object is submerged in fluid (liquid or gas), the fluid exerts an upward force on the object that is exactly opposite to the direction of gravity's pull (Fig. 1). This upward force is called the buoyant force. It is determined by the following formula

$$
\begin{equation*}
F_{w}=g \cdot \rho_{w} \cdot V \tag{5}
\end{equation*}
$$

where: $F_{\mathrm{w}}=$ buoyant force $[\mathrm{N}]$
$g=$ standard gravity $\left[\frac{m}{\mathbb{s}^{2}}\right]$
$\rho_{\mathrm{w}}=$ fluid density $\left[\frac{\mathrm{kg}}{\mathrm{m}^{\mathrm{s}}}\right]$
$V=$ fluid volume $\left[\mathrm{m}^{3}\right]$


Figure 1. Forces acting on an object immersed in a fluid.
The behaviour of an object immersed in a liquid depends on its density $\rho$ and the density of the liquid $\rho_{\mathrm{c}}$, in which it is immersed.
If the buoyant force is equal to the weight of the object $F_{\mathrm{c}}=F_{\mathrm{w}}$, then the object will float and it would be stationary. And if the density of the body is equal to the density of the liquid $\rho=\rho_{\mathrm{c}}$, then the body will float.
If the buoyant force is less than the weight of the object $F_{\mathrm{c}}>F_{\mathrm{w}}$, then the object will sink. And if the density of the body is greater than the density of liquid $\rho>\rho_{\mathrm{c}}$, then the body will sink.
If the buoyant force is greater than the weight of the object $F_{\mathrm{c}}<F_{\mathrm{w}}$, the object moves up. And if the density of the object is less than the density of the liquid $\rho<\rho_{\mathrm{c}}$, then the object will suspend in the liquid.

We can measure the object density using the Archimedes' Principle. It is the fundamental law of hydrostatics: The buoyant force on an object immersed in a fluid (liquid or gas) equals the weight of the fluid it displaces.
This principle is named after the Greek mathematician physicist, and inventor Archimedes.
According to legend, Archimedes, was asked by King Hieron II to check whether the crown was really made from pure gold. Archimedes solved the problem when he took a bath. He filled a bathtub and noticed that water spilled over the edge as he got in and he realized that the water displaced by his body was equal to the weight of his body. Forgetting that he was undressed, he went running naked down the streets shouting "Eureka!" (in Greek "I have found (it)!"). He placed the crown in a vessel of water, and then a piece of pure gold (the same as the goldsmith had received from the king). Comparing the replaced volumes of liquid, he proved that the goldsmith was a fraud.
It turned out that the crown displaced more liquid than an equal weight of gold bar. It means, that the crown had larger volume as well as lower density, and therefore it was not pure gold.

## Performance of the task

## Determining the volume of the tested objects (cylinder, cuboid). The tutor determines the method of volume measuring.

## METHOD I

- Pour water into a measuring cylinder, read its volume on the scale, and fill in the table with value as an initial volume $V_{\mathrm{p}},\left[\mathrm{m}^{3}\right]$. Remember to convert units from $\mathrm{cm}^{3}$ to $\mathrm{m}^{3}$. Pour in enough water so that, after immersing the object, the level does not exceed the scale of the measuring cylinder.
- Immerse the object (cylinder, cuboid) in water, read the water level in the cylinder, and fill in the table with the value as the final volume $V_{\mathrm{k}},\left[\mathrm{m}^{3}\right]$.

Attention! Place the solid in the measuring cylinder on the vessel wall so as not to "splash" the initial volume of water.

- Calculate the volume of an object (cylinder, cuboid) from the following formula, $\left[\mathrm{m}^{3}\right]$ :

$$
V=V_{k}-V_{p}
$$

Fill in the table with a value (V). Convert the units.

## METHOD II

- Use a calliper to measure the required dimensions of the tested objects (cylinder diameter and height, cuboid - all sides) and then calculate their volume from the formula for the volume of a cylinder or cuboid, respectively. Enter the obtained volumes ( $V$ ) in the table.
- For this method, insert dashes in the table's starting volume and ending volume.


## Determination of mass and weight of the tested objects in the air.

1. Turn on the electronic analytical balance by pressing the button (1). Follow the rules described below. Do not press any buttons other than those described here. In case of any problems, contact the tutor.
2. Hang the hook on the handle attached to the bottom of the balance and tare the scale by pressing the T button (the one on the right). The balance should show 0.000 g value.
3. Hang the test sample on the hook (Scheme 1A). Weigh the tested sample in the air and fill in the table with mass $(m)$ value, $[\mathrm{kg}]$. Don't forget to convert units.


## Scheme A of experience.

4. Calculate the force of gravity $\left(F_{\mathrm{m}}\right)$ from the following formula $[\mathrm{N}]$ :

$$
F_{m}=m \cdot g
$$

## Determination of mass and weight of the tested object in water.

1. Pour the water into a flask and measure its temperature with a thermometer. Next, immerse the tested object in water (Scheme 1 B), weigh the object and fill in table with a mass $M$ value, [kg]. Remember to convert units.
(B)


Scheme B of the experiment.
2. Calculate the force of gravity $\left(F_{\mathrm{M}}\right)$ of submerged object using the following formula [ N$]$ :

$$
F_{M}=M \cdot g
$$

## Determination of buoyant force and object density

1. Calculate the buoyant $F_{\mathrm{w}}$ from the following formula $[\mathrm{N}]$ :

$$
F_{w}=F_{m}-F_{M}
$$

2. Calculate the density of the tested samples $\rho_{\mathrm{m}}$ from the following formula, $\left[\frac{\mathrm{kg}}{\mathrm{m}^{\mathrm{s}}}\right]$ :

$$
\rho_{m}=\frac{m}{V}
$$

3. In the Physical Database, find the substance with a density close to that calculated and enter the name of the material from which the tested cuboid is made. The cylinder has been prepared especially for this exercise and is not a homogeneous material. Don't look for a material name for the cylinder!

## Determination of the water density and the theoretical value of the buoyant force.

1. Calculate the water density $\rho_{H_{2} \mathrm{O}}$ from the calculated buoyant force according to the formula and compare with the theoretical value from the Physical Database, $\left[\frac{\mathrm{kg}}{\mathrm{m}^{\mathrm{s}}}\right]$. Pay attention to the temperature!

$$
\rho_{\mathrm{H}_{2} \mathrm{O}}=F_{w} /(V \cdot g)
$$

2. Calculate the theoretical buoyant force $F_{\mathrm{T}}$ using the following formula $[\mathrm{N}]$ :

$$
F_{T}=V \cdot \rho_{T w} \cdot g
$$

The value of $\rho_{T w}$, please take from the Physical Database.

## Calculation of the uncertainties

We calculate the relative error of density measurement in the case of cylinder and cuboid from the following formula:

$$
\frac{\Delta \rho_{m}}{\rho_{m}}=\frac{\Delta m}{m}+\frac{\Delta V}{V}
$$

Where: $\Delta \mathrm{V}$ is equal $2 \mathrm{~cm}^{3}$ for the method I, and for $\frac{\Delta V}{V}$ is equal $2 \%$ for the method II, $\Delta m$ is the analytical balance accuracy. Please ask the teacher.
In the conclusions, write the result with an error for the density of the cylinder and the cuboid. For example, the density of a cylinder is $\left(\rho_{m} \pm \Delta \rho_{m}\right) \frac{\mathrm{kg}}{\mathrm{m}^{\mathrm{s}}}$.

The relative error for the buoyant force is calculated from the following formula:

$$
\frac{\Delta F_{w}}{F_{w}}=\frac{\Delta m}{m}+\frac{\Delta M}{M}
$$

where: $\Delta m$ and $\Delta M$ are the analytical balance accuracy.
In your conclusions, write down the percentage errors of buoyant for the cylinder and the cuboid.
For example, the percentage error of the buoyant force for a cuboid is $\frac{\Delta F_{w}}{F_{w}} \cdot 100 \%$ thus, it can be written as $B_{\%}=\frac{\Delta F_{w}}{F_{w}} \cdot 100 \%$.

The relative error of water density measurement (perform calculations only for the cylinder) is calculated from the following formula:

$$
\frac{\Delta \rho_{H_{2}} o}{\rho_{H_{2}} O}=\frac{\Delta F_{w}}{F_{w}}+\frac{\Delta V}{V}
$$

In the conclusions, write down the result with the error for the water density.
For example, the density of water is $\left(\rho_{\mathrm{H}_{2} \mathrm{o}} \pm \Delta \rho_{\mathrm{H}_{2}} \mathrm{o}\right) \frac{\mathrm{kg}}{\mathrm{m}^{\mathrm{s}}}$.
Compare this result with the theoretical value of water density (at a certain temperature similar to your experiment).
Is the result at the error limit close to the theoretical water density value?

The calculation results of measurement errors should be rounded up. The number of significant error digits should be the same as the measured physical quantity.

## Questions for discussion

1. What is the difference between body mass and body weight?
2. Will the buoyant force of the same mass change when we replace water with a liquid with a density greater than that of water?
