First name
Last name

Date
Degree program name

## Exercise 368:

## Determination of the wavelength of light by Newton's ring method

Graduation marking of the measuring eyepiece

| $\mathrm{x}_{1}$ | $\mathrm{x}_{2}$ | $K=x_{2}-x_{1}$ | $\alpha=\frac{1000 \mu m}{K}$ |
| :---: | :---: | :---: | :---: |
|  |  |  |  |

## Measurement of radii of interference rings

| n |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{p}_{\mathrm{i}}$ |  |  |  |  |  |  |  |  |  |
| $\mathrm{l}_{\mathrm{i}}$ |  |  |  |  |  |  |  |  |  |
| $\mathrm{a}[\mu \mathrm{m}]$ |  |  |  |  |  |  |  |  |  |
| $\mathrm{a}^{2}\left[\mu \mathrm{~m}^{2}\right]$ |  |  |  |  |  |  |  |  |  |

The color of light:

## Determination of the wavelength

| $\mathrm{b}\left[\mu \mathrm{m}^{2}\right]$ | $\mathrm{R}[\mathrm{mm}]$ | $\lambda[\mu \mathrm{m}]$ | $\Delta \lambda[\mu \mathrm{m}]$ |
| :---: | :---: | :---: | :---: |
|  |  |  |  |

## Exercise 368: Determination of the wavelength of light by Newton's ring method

## Introduction

Visible light is electromagnetic radiation, ie. a disturbance of the electromagnetic field propagating in space, to which the human eye reacts. Visible light has a wavelength range from 400 nm (violet start) to 700 nm (red end). Light also includes infrared and ultraviolet radiation. The wavelength $\lambda$ is equal to the distance between the points in space where the wave is in the same phase. In the case of electromagnetic waves, this means that the vectors of the electric field strength at points distant by a wavelength have the same direction, value, and meaning, ie., they are identical. The same is true for magnetic induction vectors. The time T, which a wave needs to travel equal to the wavelength, is called the wavelength period, while the frequency of the $f$ wave is the number of wavelengths along the path passed by the wave per unit of time (frequency is expressed in units of hertz $(\mathrm{Hz})$ which is equivalent to one (event) per second):

$$
\lambda=c \cdot T=\frac{c}{f}
$$

$$
\mathrm{c} \text { - speed of light (in a vacuum } 300000 \mathrm{~km} / \mathrm{s} \text { ) }
$$

Light has a dual wave-particle nature. It is assumed that light is a kind of stream of peculiar particles (corpuscles), called photons, which exhibit wave properties. The wave nature of light is indicated by phenomena such as diffraction and interference of light rays. Diffraction is the bending of the straight rays at the edges of the diaphragms. Wave interference is the superposition of waves of the same frequency, causing an increase or decrease in the intensity of the resulting wave - this is possible when the waves are coherent with each other, i.e. the phase difference of these waves is constant in time.

Suppose that from two sources Z1 and Z2 (Fig. 1) two identical and coherent waves emit light of wavelength $\lambda$ :


Fig. 1

For the intensity of the resultant wave to be amplified at point P , both waves must be in the same phase at this point, which will occur if the difference in paths $\Delta \mathrm{r}$ traveling by the waves from sources Z 1 and Z 2 to point P is an integer multiple of the wavelength:

$$
\Delta r=r_{2}-r_{1}=n \lambda \quad \mathrm{n}=1,2,3, \ldots \ldots
$$

Rays meeting in opposite phases will cancel each other out. Extinction of light at point P will be observed when the difference $\Delta \mathrm{r}$ of the paths of the two rays is equal to an odd multiple of the wavelength:

$$
\begin{equation*}
\Delta r=r_{2}-r_{1}=(2 n+1) \frac{\lambda}{2} \tag{1}
\end{equation*}
$$

Separate light sources are not compatible with each other. Coherent waves are produced artificially by superimposing rays coming from the same source but traveling through different optical paths. One way to obtain a path difference is through an optical system that allows you to observe Newton's rings.

Newton's rings (Fig. 2) are obtained when monochromatic (single-color) light falls on a system consisting of a plano-convex lens S and a plane-parallel plate P .



Fig. 3

Light incident perpendicularly on the lens-plate system is partially reflected on each boundary surface. Newton's rings are formed as a result of the interference of the ray reflected from the upper surface of the plate P with the ray reflected from the spherical surface of the lens S . The interference results are light circles (the waves will reinforce (add) - constructive interference) and dark circles (the waves will cancel (subtract) and the resulting light intensity will be weaker or zero - destructive interference. Their position depends on the thickness of the air layer between the plate and the lens, as changing the thickness of the air layer also changes the path difference of the interfering rays. The following rings are assigned row numbers, with the middle dark circle row 0 , the smallest dark ring row 1 , the next dark ring row 2 , etc.

The thickness of the air layer can be estimated using the laws of geometry applied to the schematic layout shown in Fig. 3. The similarity of the ACE and CDE triangles results in the following proportion:

$$
\begin{equation*}
\frac{e}{a}=\frac{a}{2 R-e} \tag{2}
\end{equation*}
$$

where e-thickness of the air layer, a - radius of the base of the spherical cap (also the radius of the ring), $R$ - radius of curvature of the lens. Since $2 R \gg e$, after simplifying and transforming formula (2), we get:

$$
\begin{equation*}
e=\frac{a^{2}}{2 R} \tag{3}
\end{equation*}
$$

The condition of extinguishing the light waves requires that the path difference of the interfering rays be an odd multiple of the wavelength (formula 1). The difference in paths $\Delta \mathrm{r}$ of the ray reflected from the plate P and the ray reflected from the spherical surface of the lens S is equal to:

$$
\begin{equation*}
\Delta r=2 e+\frac{\lambda}{2} \tag{4}
\end{equation*}
$$

$\lambda / 2$ is the result of the phase change of the wave reflected from the plate surface P . When the wave is reflected from an environment with a higher refractive index than the refractive index of the environment in which the wave is traveling, there is a phase change of $180^{\circ}$, which corresponds to the difference in the path equal to $\lambda / 2$.
After substituting formula (4) to formula (1), we obtain the equality:

$$
\begin{equation*}
2 e+\frac{\lambda}{2}=(2 n+1) \frac{\lambda}{2} \tag{5}
\end{equation*}
$$

and after substituting formula (3) to formula (5) and transforming it, we get the formula for the radius of the dark ring of the order n :

$$
\begin{equation*}
a^{2}=n \lambda R \tag{6}
\end{equation*}
$$

Knowing the radius of curvature of the lens R and the radius of the ring of the order n , the wavelength of the light can be calculated from this formula.

## Performance of the task

The scheme of the measuring system is shown in Fig. 4:


Fig. 5
A parallel beam of monochromatic light from the Z source, after partial reflection from the glass plate P set at an angle of $45^{\circ}$ to the optical axis of the microscope, hits the lens-plate system. The rays reflected upwards pass through the P plate and go to the microscope objective and then to the observer's eye. A light-emitting diode is used as a light source, which emits light with a narrow wavelength range. The diode, lens, plane-parallel plate, and plate set at an angle of $45^{\circ}$ are permanently connected in a single experimental set-up (Fig. 5).
To measure the radii of interference rings, we use a microscope with a low magnification and a measuring eyepiece. The measuring eyepiece screw drum is divided into 100 divisions. Inside the eyepiece, there is the main scale of the eyepiece (numbers from 0 to 8 ) and a cross made of thin threads, which move during the rotation of the screw. We start the measurements by marking the eyepiece scale.

## Marking of the graduation of the measuring eyepiece

1. Place the metal plate on the microscope table with a small hole in the center. A calibration slide is placed on the plate. A small circle is marked on the slide, inside which a micrometric scale is drawn ( 1 mm divided into 100 segments). Place the slide exactly under the microscope eyepiece and illuminate it with the light reflected from the microscope mirror. When looking at the eyepiece, find the micrometric scale by adjusting the depth of field with the knob on the side of the microscope.
2. Place the cross made of "spider threads" on the first line of the micrometric scale (this line corresponds to the value 0 ) (fig. 6):


Fig. 6
Above the "spider cross", you can see a double vertical line that moves along with the cross along a series of numbers from 0 to 8 . We read the figure lying to the left of the double line. This digit represents the number of hundreds (i.e. $0,100,200,300$, etc. up to 800 ). Read the number of tens and ones on the barrel of the measuring eyepiece. We enter the entire value in the table as x 1 .
3. By turning the eyepiece screw, set the cross on the last micrometer scale line (this line corresponds to 1 mm ). We read the value from the measuring eyepiece as $\mathrm{x}_{2}$.
4. The number of graduations on the measuring eyepiece barrel per $1 \mathrm{~mm}(\mathrm{~K})$ is the difference between the readings for the 0 and 1 mm lines. We calculate:

$$
\begin{equation*}
K=x_{2}-x_{1} \tag{7}
\end{equation*}
$$

5. If 1 mm is equal to K divisions of the measuring eyepiece, then the value of the smallest division of the measuring eyepiece barrel expressed in $\mathrm{w} \mu \mathrm{m}$ will be equal to:

$$
\begin{equation*}
\alpha=\frac{1000 \mu m}{K} \tag{8}
\end{equation*}
$$

## Measurement of radii of interference rings

The numbers of the row of the rings, for which the measurements are made, are determined by the instructor.

1. Place the experimental set from fig. 5 under the lens. Connect the diode to an electrical socket. Under the microscope, you should see a uniform background in the color of the Light-emitting diode (LED).
2. By adjusting the depth of field of the microscope, find Newton's rings. After finding, place the rings in the center of the field of view (the center of the rings is approximately under the number 4 of the eyepiece scale, Fig. 7a).


Fig. 7a
Fig. 7b
3. The radius of the $n$-ring is half the diameter of the n-ring. To measure the diameters of the rings, we take readings of the position of the selected rings to the right and left of the center. We start from
the right side. Place the micrometer screw on the dark ring of a given row (the measurement of the ring of row 1 is shown in Fig. 7b) and read the indications of the measuring eyepiece. We enter as $\mathrm{p}_{\mathrm{i}}$.
4. Move the cross from «spider threads» to the dark ring of the next row. (We select $8-9$ rings for measurements. They do not have to be consecutive rings, for example, you can choose rings: $1,2,3,5,7,9,10,12$ ). We take a reading, enter it in the table, and then measure the positions of the next dark rings to the right of the center. After completing the measurements on the right side, we make the same measurements on the left side of the rings. When taking measurements, be very careful not to move the experimental set-up to the microscope stage (if it moves, start the measurements all over again).
5. We calculate the radius of the rings by converting the scale of the measuring eyepiece drum into micrometers:

$$
\begin{equation*}
a_{i}=\frac{1}{2}\left(p_{i}-l_{i}\right) \cdot \alpha \tag{9}
\end{equation*}
$$

6. We calculate $a_{i}^{2}$.

## Wavelength graph and calculation

Based on the measurement data, a graph should be drawn up $a_{i}^{2}=f(n)$. This relationship is a linear function of the equation $y=a+b x$. The comparison of this equation with the formula (6) shows that $b=\lambda \cdot R$. Hence:

$$
\begin{equation*}
\lambda=\frac{b}{R} \tag{10}
\end{equation*}
$$

R is the radius of curvature of the lens. The value of the radius of curvature of the lens should be taken from the plug of the experimental set-up. Reading the relevant data from the chart can be done in two ways: manually or by using a spreadsheet (eg Microsoft Office Excel, OpenOffice Calc).

## Manual way

1. We draw the $a_{i}^{2}=f(n)$ dependence of the square of the radius of the ring on the row of the ring on graph paper.
2. Fit the straight line $y=a+b x$ to the marked measurement points. We mark two points on the straight line Fig. 8 cannot be measurement points, points should be selected as close as possible to the beginning and end of the straight line. From the x and y axes we read the coordinates of the selected points $\left(n_{i}, a_{i}^{2} ; n_{j}, a_{j}^{2}\right)$ :


Fig. 8
(For the inquisitive: according to the formula (6), the straight line should pass through the point (0,0), i.e. $a=0$. However, in reality, due to the slight flattening of the lens in the area of contact with the flat plate, the straight line does not have to pass through the point $(0,0), a \neq 0$.
3. Calculate the slope of the line from the formula: $b=\frac{a_{i}^{2}-a_{j}^{2}}{n_{i}-n_{j}}$
4. Calculate the wavelength from the formula (10). (NOTE THE UNITS! $\mu m \leftrightarrow m m \leftrightarrow m$ )

## Using an Excel spreadsheet (it's very similar to OpenOffice)

7. In the sheet, in the first column, write the row of the ring n , in the second - the square of the ring radius $\mathrm{a}^{2}$. We select cells with numbers. In the Menu, we choose to Insert a chart. Select the scatter plot without lines and close the chart selection window with the Finish button. Right-click any measurement point on the chart and select Add trend line. We choose Linear type, and in Options, we select Display equation on the chart. We close the window with the $O K$ button (in OpenOffice the trendline is called a regression curve)
8. The function of the form $y=b x+a$ appears on the graph. We write the value of $b$ which is the slope of the line.
9. We calculate the wavelength from the formula (10).

## Calculation of the uncertainties

The error in determining the wavelength consists of: the accuracy of marking the scale of the measuring eyepiece drum $\Delta \alpha$, the accuracy of reading the position of the rings $\Delta \mathrm{p}$ and $\Delta \mathrm{l}$ and determining the radii $\Delta \mathrm{R}$, the accuracy of fitting the straight line to the measurement points and determining the slope $\Delta \mathrm{b}$, and the accuracy of the radius of curvature $\Delta \mathrm{R}$ estimation. It can be assumed that:
$\frac{\Delta \alpha}{\alpha}=\frac{\Delta l}{l}=\frac{\Delta p}{p}=\frac{\Delta R}{R}=0,5 \%$
$\frac{\Delta a}{a}=\frac{\Delta \alpha}{\alpha}+\frac{\Delta l}{l}+\frac{\Delta p}{p}$
$\frac{\Delta b}{b}=2 \frac{\Delta a}{a}+5 \% \quad(5 \%$ is related to the accuracy of reading the data from the graph)
To sum up, the accuracy of the determination $\lambda$ is calculated from the formula:
$\frac{\Delta \lambda}{\lambda}=\frac{\Delta b}{b}+\frac{\Delta R}{R}$

We calculate the value of $\Delta \lambda$.

## Question for discussion

What is the tabular range for the color used in the experiment?
Does the tabular range for the color table coincide with the designated $\lambda \pm \Delta \lambda$ range?

