First name .... Date ....

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Exercise 362

Determination of the focal length of lenses by the Bessel method and measurement of radii of curvature of the lens using a spherometer

I. Determining the focal length of a converging and diverging lens

The distance of the object from the screen	$l =$

Type lenses	Distance between the lens and the screen [m]				Difference in distance between images and a lens	Focal length
	Enlarged image		Reduced image			
	b_{1i} i=1, 2, 3	average b_1	b_{2i} i = 1, 2, 3	average b_2	$d = \lvert b_1 - b_2 \rvert$	[m]
Converging Focusing						$f_1 =$
System of lenses						$f_u =$

Focal length of the diverging lens, [m]	$f_2 =$

II. Measurement of radii of curvature of the lens and determination of the refractive index

length of the sides of the triangle c_i [m]	average value c [m]	Zero position h_0 [m]

Spherometer measurement results

Lens	Surface I			Surface II			n
	h_1 [m]	$h_1 - h_0$ [m]	R_1 [m]	h_2 [m]	$h_2 - h_0$ [m]	R_2 [m]	
converging							
diverging							

Exercise 362. Determination of the focal length of lenses by the Bessel method and measurement of radii of curvature of the lens using a spherometer

Lenses

Let us first consider a doubly convex lens, bounded by two spherical surfaces with radii of curvature R_1 and R_2. For example, suppose that several rays of light approach the lens and that the rays move parallel to the *principal axis* (Fig.1). On reaching the lens's front surface, each light ray refracts in the direction normal to the surface. The light ray passes from the air into a denser medium at this boundary. Then, as the light ray passes from a medium in which it is moving fast (less optically dense) to a medium in which it is moving relatively slower (more optically dense), it will refract towards the normal line. As a ray of light refracts across the boundary and enters the lens, it travels in a straight line up to the back surface of the lens. At this boundary, each ray of light refracts from normal to the surface. Then, as a ray of light passes from a medium in which it is moving slowly (more optically dense) to a medium in which it is moving fast (less optically dense), it deflects from the normal line.

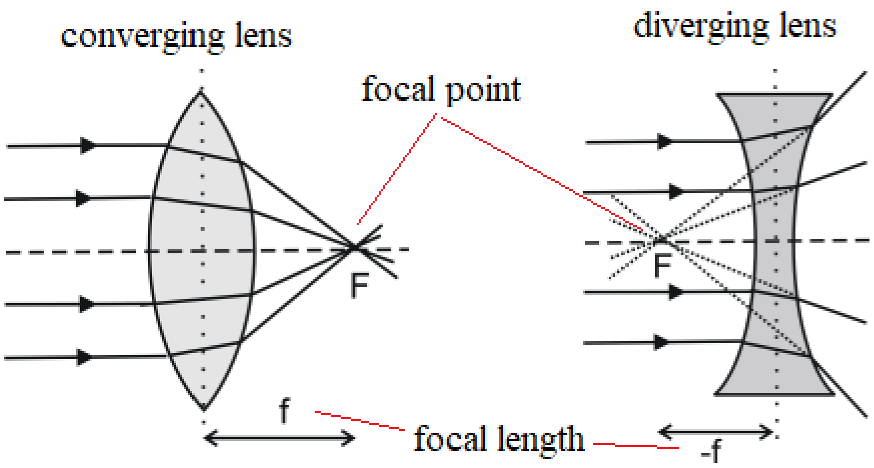

Fig. 1a Fig. 1b

The figure above (Fig. 1a) shows the behaviour of rays incident on a lens parallel to the principal axis. Note that the two rays converge at a single point; this point is called the *focal point* of the lens. The figure (Figure 1b) shows the behaviour of incident rays parallel to the principal axis of a double concave lens. As with the biconvex lens, light deflects towards the normal as it enters the lens and away from the normal as it exits the lens. However, due to the different shape of the biconvex lens, the incident rays do not converge to a point when refracted through the lens. Instead, when refracted through the lens, the incident rays diverge. For this reason, a double-concave lens can never produce a true image. Doubly concave lenses produce virtual images. If the refracted rays are extended backwards behind the lens, we make an important observation. The extension of the refracted rays will intersect at a certain point. This point is called the focal point. Diverging lens, such as this double concave lens, does not focus the incident light rays that are parallel to the principal axis but diverges them. For this reason, a diverging lens is said to have a negative focal length.

Lens Equations

The focal length of a lens depends on the refractive index n of the material from which it is made and its radii of curvature R_1 and R_2. In the case of thin lenses, which are the subject of our discussion, someone can calculate the focal length from the *lens formula*:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{1}$$

which is often called the lens *maker's equation*. The radius of curvature of the convex surface is positive, in turn the radius of the concave surface is negative. Since the focal length of the scattering lens is negative, the sum of the reciprocal of the radii of curvature for this type of lens must also be negative. The reciprocal of the focal length $D = 1/f$ is called the *optical power* (also referred to as dioptric power, refractive power, focusing power, or convergence). The unit of the optical power is the diopter, [D]; 1D = $1m^{-1}$.

We shall consider only the particular case of a thin lens, a lens in which the thickest part is thin relative to the object distance a, the image distance b, and the radii of curvature R_1 and R_2 of the two surfaces of the lens. We can prove that for rays which make small angles with the central axis, a thin lens has a focal length of f. Moreover, a and b are related to each other by:

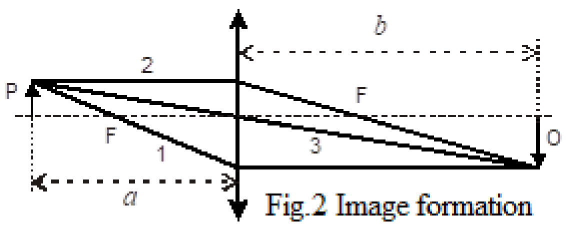

Fig.2 Image formation

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f} \tag{2}$$

The focal length of an optical system composed of two thin lenses with focal lengths f_1 and f_2, put together, satisfies the dependence

$$\frac{1}{f_u} = \frac{1}{f_1} + \frac{1}{f_2} \tag{3}$$

Determination of the focal length of the lens by the Bessel method

Converging lens

In this method (see Fig.3), we set a constant distance between the object and the screen *l*, and then we find two positions of the lens, when the enlarged (position 1) and reduced (position 2) image which is created on the screen. In both cases, we measure the distance between the lens and the object x_1 and x_2 and calculate the difference in the positions of the lens *d*. The object distance in lens position one x_1 is equal to the image distance in lens position two y_2 ($x_1=y_2$), and the object distance in lens position two x_2 is equal to the image distance in position on y_1 ($x_2=y_1$).

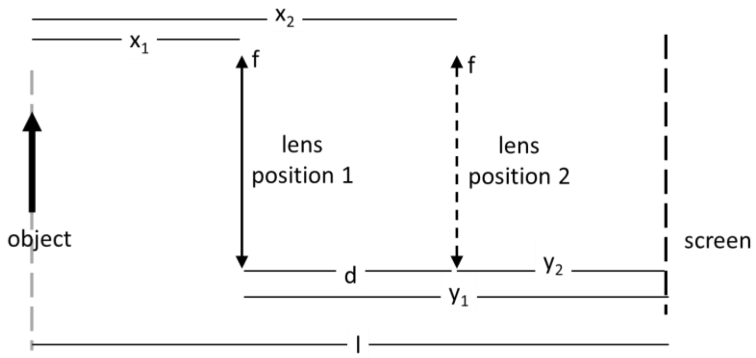

Fig.3

The lens formula for the enlarged and reduced image is as follows:

$$\frac{1}{f}=\frac{1}{x_1}+\frac{1}{y_1}=\frac{1}{x_2}+\frac{1}{y_2}=\frac{1}{x_1}+\frac{1}{x_2}$$

$$l=x_1+y_1=x_2+y_2=x_1+x_2$$

$$x_2=l-x_1,$$

$$d=x_2-x_1=l-x_1-x_1=l-2x_1$$

$$x_1=\frac{l-d}{2}$$

$$x_2=l-\frac{l-d}{2}=\frac{l+d}{2}$$

$$\frac{1}{f}=\frac{1}{\frac{l-d}{2}}+\frac{1}{\frac{l+d}{2}}=\frac{4l}{l^2-d^2}$$

Finally, we get the formula for the focal length:

$$f=\frac{l^2-d^2}{4l} \qquad (5)$$

Diverging lens

Since diverging lenses do not produce real images, we combine a diverging lens with a focal length f_2 with a converging lens with a known focal length f_1 into a lens system that should have the properties of a converging lens – it is possible when $|f_2| > f_1$. Then, in the same way as for a single converging lens, we determine the focal length f_u of the system.

By transforming the dependence (3), we obtain the formula

$$f_2=\frac{f_1\cdot f_u}{f_1-f_u} \qquad (6)$$

from which, after substituting the focal lengths f_1 and f_u determined by the Bessel method, we calculate the focal length of the diverging lens f_2 .

Determination of the radius of curvature of the lens using a spherometer

The height h of a spherical lens can be measured using a spherometer. The essential measuring element of the spherometer is a movable vertical micrometre screw or dial micrometre sensor. The measuring element is mounted on tripod support.

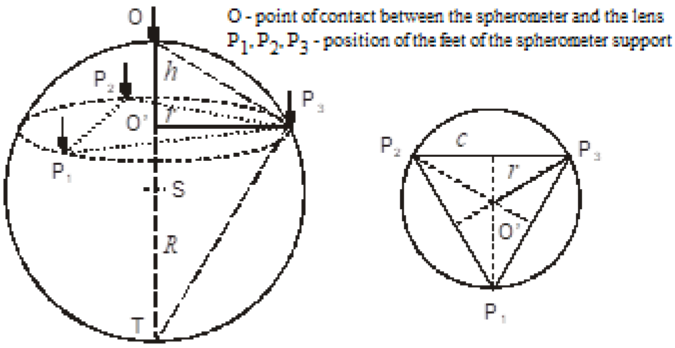

Fig.4 Measurement of the radius of curvature

The conically pointed feet of the spherometer form the vertices of an equilateral triangle, through the centre of which passes the axis of the screw (right side of Fig.4).

Considering the OP_3T triangle shown in Fig. 4 (left side), whose right angle is based on the diameter, we obtain a relationship between the radius R of the sphere (in our case, the radius of curvature of the lens), *r* - the radius of the circle forming the base of the spherometer; and *h* – height (look at fig.4) :

$$r^2 = (2R - h) \cdot h \tag{7}$$

The circle forming the base of the spherometer is a circle inscribed in an equilateral triangle with side c formed by the base of the spherometer. So there is a relationship:

$$r = \frac{c}{\sqrt{3}},$$

Hence, we get the formula for the radius of curvature:

$$R = \frac{c^2}{6h} + \frac{h}{2}. \tag{8}$$

So by measuring c and h, we can find the radius of curvature. The height of h is the difference between the indication of the spherometer set on a flat surface and the indication read when the spherometer is set on the one of the surfaces of the lens under test.

We assume h> 0 for a convex surface and h <0 for a concave surface.

The refractive index of the lens material

After determining the lens's focal length and the radii of curvature of its surface, we can calculate the refractive index of the lens material, so we get :

$$n = \frac{R_1 R_2}{f(R_1 + R_2)} + 1 \tag{9}$$

Performance of the task

Determination of the focal length of the converging and diverging lens using the Bessel method

1. Read the distance l of the object from the screen on the optical bench.
2. We are looking for a position b_1 of the lens corresponding with a sharp magnified image on the screen and then a position b_2 corresponding to the sharp reduced image.
3. Repeat no2. three times and calculate the mean values - their difference gives the distance d: $d = b_1 - b_2$
4. Calculate the focal length f_1 of the lens based on the Bessel formula (5).
5. Connect a scattering lens with an unknown focal length f_2 with a converging lens with an already determined focal length f_1. We determine, using the above method, te focal length f_2 of the system.
6. Connect the diverging lens of an unknown focal length of f_2 with a converging lens with a calculated focal length of f_1. Then, let's determine the system's focal length of f_u using the above method.
7. Calculate the focal length f_2 using the formula (6).

Determination of the radius of curvature of the lenses

1. Imprint the marks of the legs of the spherometer tripod on a sheet of paper. Apply the calliper blades to the traces on the paper and read the lengths of the triangle's three sides. Calculate the average value of the obtained results as the value of the side of the equilateral triangle — c.
2. Determine the zero position of the spherometer. To do this, place the spherometer on a smooth plate and read its indication (the tip of the spherometer sensor should touch the plate).
3. Then, put the spherometer on the convex surface of the converging lens and read its indication of h_1. Calculate the difference $h = h_1 - h_0$.
4. We carry out the same measurements on the other surface of the lens – h_2, which can also be convex or concave. In the latter case, the value $h = h_2 - h_0$ is negative.
5. Put the values of h and c into formula (8) and then calculate the radii of curvature R1 and R2 of the converging lens.
6. Put the focal length value determined by the Bessel method into the formula (9) to calculate the refractive index.
7. Repeat the same steps to the diverging lens.

Calculation of the uncertainties

The maximum absolute errors of the measured quantities are calculated using differential calculus. We make calculations for the **converging lens.**

Measurement error of focal length

$$f_1 = \frac{l^2 - d^2}{4l}, \qquad \Delta f_1 = \left|\frac{\partial f_1}{\partial l}\right| \Delta l + \left|\frac{\partial f_1}{\partial d}\right| \Delta d = \frac{l^2 + d^2}{4l^2} \Delta l + \frac{d}{2l} \Delta d .$$

Because $\Delta l = \Delta d$=2 mm, formula for Δf_1 can lead to a simpler form:

$$\Delta f_1 = \frac{(l+d)^2}{4l^2} \Delta l . \tag{10}$$

Measurement error of radius of curvature

$$R = \frac{c^2}{6h} + \frac{h}{2}, \qquad \Delta R = \left|\frac{\partial R}{\partial c}\right| \Delta c + \left|\frac{\partial R}{\partial h}\right| \Delta h .$$

After calculating the partial derivatives:

$$\Delta R = \left|\frac{c}{3h}\right| \Delta c + \left|-\frac{c^2}{6h^2} + 0,5\right| \Delta h . \tag{11}$$

The value Δh equals twice the accuracy of a spherometer (2*0,01 mm=0,02 mm).

$\Delta c = \max\left|c - c_i\right| + 0{,}1\text{mm}$, i = 1, 2, 3; (0,1 mm is the accuracy of the caliper)

Measurement error of refractive index

$$n = \frac{R_1 R_2}{f_1 (R_1 + R_2)}, \qquad \Delta n = \left|\frac{\partial n}{\partial R_1}\right| \Delta R_1 + \left|\frac{\partial n}{\partial R_2}\right| \Delta R_2 + \left|\frac{\partial n}{\partial f_1}\right| \Delta f_1 .$$

The calculations of partial derivatives lead to the formula:

$$\Delta n = \frac{R_2^{\,2} \cdot \Delta R_1 + R_1^{\,2} \cdot \Delta R_2}{|f_1| \cdot (R_1 + R_2)^2} + \left|\frac{R_1 \cdot R_2}{R_1 + R_2}\right| \cdot \frac{\Delta f_1}{f_1^{\,2}} . \tag{12}$$

Δf_1, ΔR_1, ΔR_2, — from the calculations above (formulas 10 and 11).