First name .... Date ....

Last name .... Degree program name ....

Exercise 361

Study of two-lens system

Table I: Determining Focal Length of the Lens

Distance between the object and the screen (60 cm – 110 cm)	$l =$			
Lens	Enlarged image Distance between the object and the lens	Reduced image Distance between the object and the lens	$d = x_2 - x_1$	Focal length $f = \frac{l^2 - d^2}{4l}$ Δf - uncertainty
1	$x_1 =$ image:…………..…	$x_2 =$ image:……..……….	$d =$	$f_1 =$ $\Delta f_1 =$
2	$x_1 =$	$x_2 =$	$d =$	$f_2 =$ $\Delta f_2 =$

Table II: Determining position of the image in two-lens system

A. Lenses spaced apart by a distance

Distance between the object and the lens *1* (25 cm – 35 cm)	$x_1 =$	
Distance between the lenses (40 cm – 55 cm)	$s =$	
Distance between the image and the lens *2*	Measured value	Calculated value $\frac{1}{y_2} = \frac{1}{f_2} - \frac{1}{s - \frac{f_1 x_1}{x_1 - f_1}}$ Δy_2 - uncertainty
image:……………………………… ……………………………………..	$y_2' =$	$y_2 =$ $\Delta y_2 =$

B. Lenses spaced apart by a minimal distance

Distance between the image and the lens *2*	Measured value	Calculated value $\frac{1}{y_2} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{1}{x_1}$ Δy_2 - uncertainty
image:……………………………… ……………………………………..	$y_2' =$	$y_2 =$ $\Delta y_2 =$

Exercise 361: Study of two-lens system

Purpose

The purpose of this laboratory activity is to study the images formed by the converging lenses.

Theory

A spherical lens is an optical device usually made of glass or transparent plastic with two spherical surfaces, which can be concave or convex. When the light ray passes through the lens it is refracted at the both surfaces. Refraction is caused by a change in speed of light, when it enters the medium of different optical density (different index of refraction). The direction of the refracted ray depends on whether the ray enters an optically denser or thinner medium. For example glass has a higher optical density than air. When the light ray passes from air to glass it refracts toward the normal (Fig.1), when the light ray passes from glass to air, it refracts deviating from the normal. A normal is the line drawn perpendicular to the medium surface at the point of incidence. The curvature of the surface causes the refracted ray to run in the opposite direction to the principal axis for the concave surface and towards the principal axis for the convex surface, when the ray is passing from the optically thinner medium to optically denser medium (Fig.1), and vice versa, when the ray goes from optically denser to thinner medium.

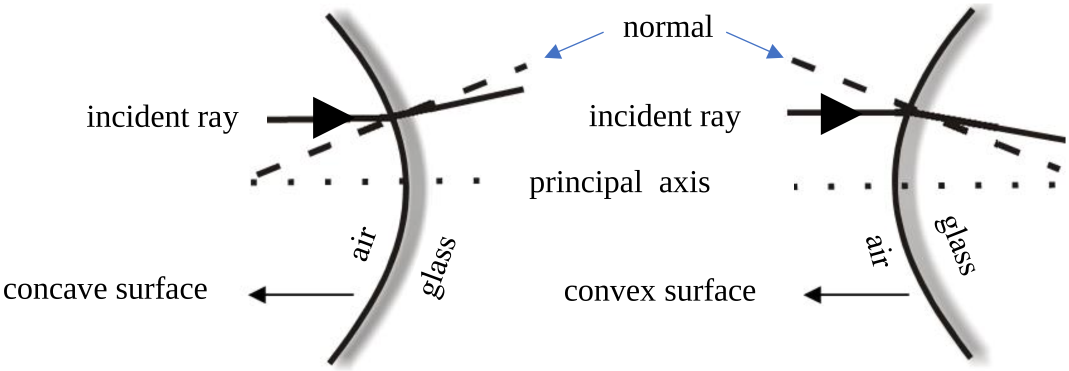

Fig.1

Based on the shape, the lenses can be grouped as convex (converging) lenses or concave (diverging) lenses. In a converging lens all incident rays parallel to the principal axis, after leaving the lens pass through one point called focus (F in Fig.2). In a diverging lens it is the extensions of the rays that intersect in the point called the apparent focus. The distance from the centre of the lens to its principal focus is called the focal length f (Fig.2). The focal length of the diverging lens is negative ($f<0$).

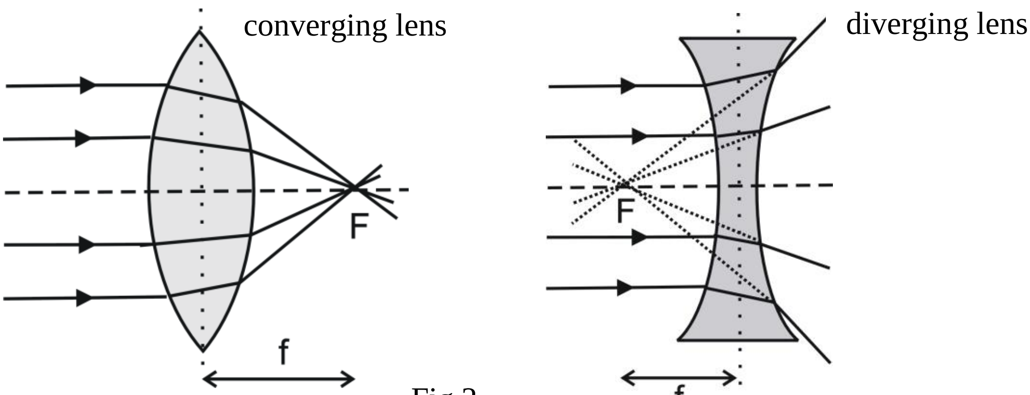

Fig.2

In ophthalmology the power of lenses used in spectacles is measured in dioptres. The optical power Z of a lens is the invers of its focal length: Z=1/f. The dioptre is the reciprocal of a metre: dioptre=1/m.

If we place an object on one side of the lens, a sharp image of that object will be created at a certain distance from the lens (Fig.3; red – object, blue – image). The image can be real or virtual. The real image is created on the other side of the lens than the object and can be seen on the screen. The virtual image is created on the same side of the lens as the object and we cannot see it on the screen. We have to look at the object through the lens to see the virtual image. We can see the image with our eyes, because the lens of the eye converges the rays into a real image projected on our retina. Diverging lenses only produce virtual images. Converging lenses produce virtual images when the object is close to the lens, just like when we look through a magnifying glass which is simply a converging lens. The object is at a distance less than the focal length then (5 and 6 in Fig.3). In other situations, the converging lenses produce real images (1-3). The image can be upright or inverted. Diverging lenses only give upright images. Converging lenses produce upright images when the object is placed close to the lens, at a distance shorter than the focal length (5 and 6). Otherwise, the images are inverted (1-3). The image can be enlarged or reduced compared to the object. Diverging lenses always reduce the image. In converging lenses, it depends on the distance between the lens and the object. A converging lens reduces when the object is far from the lens, at a distance greater than the double focal length (1). When the object is closer than the double focal length, the image is enlarged (3, 5 and 6). When the object is at a distance equal to the double focal length, the image is the same size as the object (2), and at a distance equal to the focal length, the image is not formed at all (4).

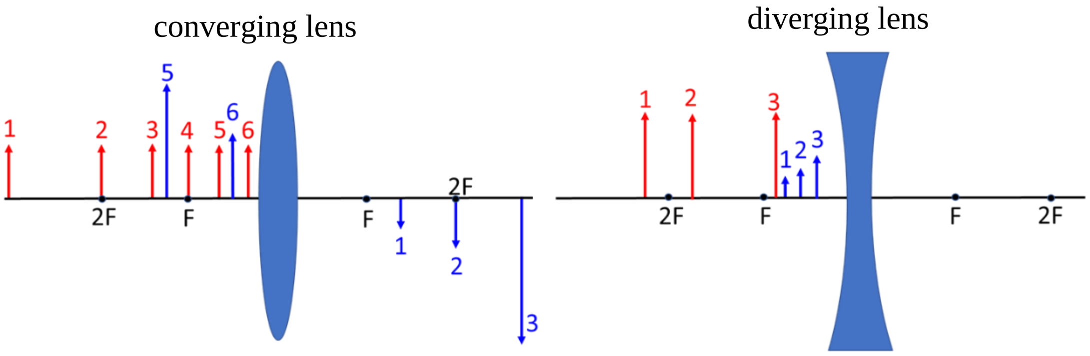

Fig.3

The focal length of the lens can be calculated from a simple lens equation:

$$\frac{1}{f} = \frac{1}{x} + \frac{1}{y}$$

by measuring the distance of an object from the lens *x* and the distance of an image from the lens *y*. In this exercise, however, we will use the more accurate Bessel method (Fig.4).

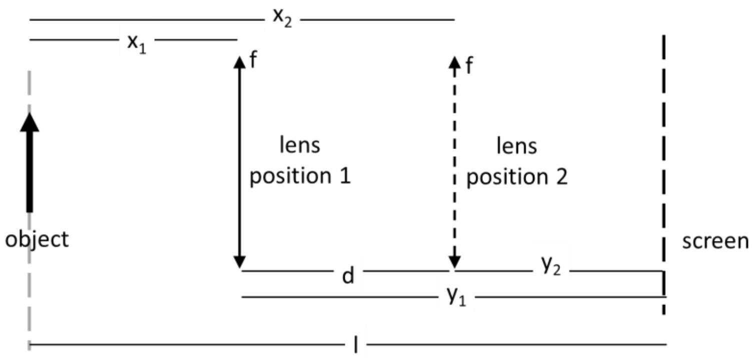

Fig.4

In this method, we set a constant distance between the object and the screen l, and then we find two positions of the lens, when the enlarged (position 1) and reduced (position 2) image is created on the screen. In both cases, we measure the distance between the lens and the object x_1 and x_2 and calculate the difference in the positions of the lens d. The object distance in lens position one x_1 is equal to the image distance in lens position two y_2 (x_1=y_2) and the object distance in lens position two x_2 is equal to the image distance in position on y_1 (x_2=y_1).

The lens formula for the enlarged and reduced image is as follows:

$$\frac{1}{f} = \frac{1}{x_1} + \frac{1}{y_1} = \frac{1}{x_2} + \frac{1}{y_2} = \frac{1}{x_1} + \frac{1}{x_2}$$

$$l = x_1 + y_1 = x_2 + y_2 = x_1 + x_2$$

$$x_2 = l - x_1,$$

$$d = x_2 - x_1 = l - x_1 - x_1 = l - 2x_1$$

$$x_1 = \frac{l - d}{2}$$

$$x_2 = l - \frac{l - d}{2} = \frac{l + d}{2}$$

$$\frac{1}{f} = \frac{1}{\frac{l - d}{2}} + \frac{1}{\frac{l + d}{2}} = \frac{4l}{l^2 - d^2}$$

Finally, we get the formula for the focal length:

$$f = \frac{l^2 - d^2}{4l}$$

Two or more lenses can be combined into one optical system. Such systems are widely used in practice, e.g. in ophthalmology. The lens of glasses with the lens of the eye form a system with a resultant focal length suitable for a sharp vision of near or distant objects. The position of the image in a two-lens system can be calculated using the lens formula. For the first and second lens the lens formula is as follows:

$$\frac{1}{f_1} = \frac{1}{x_1} + \frac{1}{y_1} \qquad \frac{1}{f_2} = \frac{1}{x_2} + \frac{1}{y_2}$$

$$\frac{1}{y_1} = \frac{1}{f_1} - \frac{1}{x_1} \qquad \frac{1}{y_2} = \frac{1}{f_2} - \frac{1}{x_2}$$

$$y_1 = \frac{f_1 x_1}{x_1 - f_1}$$

In a two-lens system, the image obtained from the first lens becomes the object for the second lens. Therefore, the distance between the lenses can be expressed as

$$s = y_1 + x_2$$

$$x_2 = s - y_1 = s - \frac{f_1 x_1}{x_1 - f_1}$$

$$\frac{1}{y_2} = \frac{1}{f_2} - \frac{1}{s - \frac{f_1 x_1}{x_1 - f_1}}$$

From this equation we can calculate y_2.

When we place the lenses very close to each other ($s \approx 0$), we can apply the formula for the lens system to calculate the distance of the image form the lenses:

$$\frac{1}{f_u} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f_u} = \frac{1}{x_1} + \frac{1}{y_2}$$

$$\frac{1}{x_1} + \frac{1}{y_2} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{y_2} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{1}{x_1}$$

From this equation we can calculate y_2.

Performance of the task

1. A light source, an object (a plate with a cut-out hole) and a screen are placed on the optical bench. Set the distance *l* between the object and the screen in the range of 60 - 110 cm and enter it in the measurement table.
2. Look at lenses *1* and *2*. How to determine the type of lens (converging lens or diverging lens)?

3. Place the lens *1* on the optical bench and move it until you get a clear magnified image. Read the distance between the object and the lens x_1. Is the image upright or inverted? Record the observation in the measurement table.
4. Find a sharp, reduced image. Read the distance between the object and the lens x_2. Is the image upright or inverted? Record the observation in the measurement table.
5. Remove the lens *1* from bench and take the same measurements with the lens *2*.
6. Calculate the focal length of both lenses. Compare them with the factory values of the lens power given on the lenses (calculate the focal length based on the power, the value measured by the Bessel method should not differ by more than 1 cm from the factory value).
7. Place both lenses on the bench. Keep the order of the lenses (from the left: object, lens 1, lens 2, screen). Set the distance x_1 between the object and the lens 1 in the range of 25 - 35 cm, and the distance *s* between the lens 1 and the lens 2 in the range of 40 – 55 cm. Record these distances in the measurement table.
8. By moving the screen, try to find the sharpest image and note its distance y'_2 from the lens *2*. Is the image upright or inverted? Is the image enlarged or reduced? Record the observation in the measurement table.
9. Calculate the distance y_2 of the image from the lens *2*.
10. Move the lens 2 to lens 1 until they touch to each other (we do not change the position of the lens 1). By moving the screen find the new position of the sharp image and note the distance y'_2 between the screen and the lens *2*. Is the image upright or inverted? Is the image enlarged or reduced? Record the observation in the measurement table.
11. Calculate the distance y_2 of the image from the lens *2*.

Calculation of the uncertainties

We perform one type of measurement: we measure distances with a ruler with a millimetre scale. Uncertainty of that measurement contribute to uncertainty of the final results. We use the following formulas to approximate the uncertainty of the focal length and the image distance:

$$\Delta f = \frac{(l+d)^2}{4l^2}\Delta l \qquad \Delta y_2 = \left(\frac{\Delta f_1}{f_1} + \frac{\Delta f_2}{f_2} + \frac{\Delta x_1}{x_1}\right) \cdot y_2$$

where Δl and Δx_1 is the uncertainty of the distance measure. Taking into account the uncertainty associated with finding a sharp image and the accuracy of the measurement, we can assume a uncertainty of distance measurement of 0,5 cm. We use the second formula for both lenses sets (for lenses that are spaced apart by a distance and for lenses at minimum distance from each other). The values of the calculated uncertainties should be rounded up to the first significant figure and entered in the appropriate places in the table.

Questions for discussion:

1. Are the measured and the factory values of the focal lengths similar or different? Use the uncertainty value for comparison.
2. Is the measured and calculated distance of the image from the lens *2* in the system of two lenses similar or different? Use the uncertainty value for comparison.
3. Is the measured and calculated distance of the image to lens *2* in the close-up system similar or different? Use the uncertainty value for comparison.
4. In a system of two lenses, by changing the distance between the lenses, you can get an upright or inverted image. Why?
5. How to determine the type of lens (converging lens or diverging lens)?
6. A Kepler telescope is used to observe distant objects. It is a two-lens system. What is the difference between the two-lens system used in the exercise and the Kepler telescope?