First name .... Date ....

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Exercise 252

Study of the transformer

I. The determination of the transformer ratio

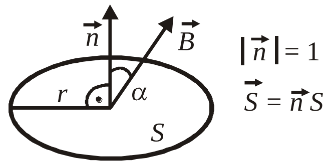
U_1 [V]							
U_2 [V]							
$k = U_2/U_1$							
Average value, $\bar{k}$							

II. The determination of a transformer efficiency

Primary circuit			Secondary circuit			η
U_1, [V]	I_1, [mA]	P_1, [mW]	U_2, [V]	I_2, [mA]	P_2, [mW]	[%]

Exercise 252. Study of the transformer

Introduction



The essence of the transformer operation is to use the *phenomenon of electromagnetic induction*. This phenomenon is based on the induction of an *electromotive force in the electric circuit* $\mathcal{E}_{ind}$, abbreviated as EMF, due to the change in time of the magnetic flux penetrating this circuit. When the circuit is closed, an induced electric current flows through it.

In the case of a homogeneous magnetic field for which the magnetic induction vector $\vec{B}$ is the same at every point, the *magnetic flux* Φ through the surface S is the dot product of the vectors $\vec{B}$ and $\vec{S}$:

$$\Phi = \vec{B} \cdot \vec{S} = B \cdot S \cdot \cos\alpha ,$$

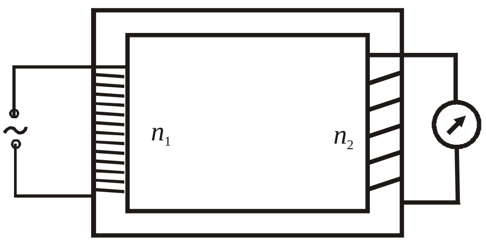
where α is the angle between the vectors $\vec{B}$ and $\vec{S}$. The vector $\vec{n}$, shown in the figure, is a unit vector, perpendicular to the surface S. The unit of magnetic flux is Weber [Wb]; 1 Wb = 1 T·m^2. Here T is the unit of magnetic induction (Tesla): $1\ \mathrm{T} = 1\,\mathrm{N}/(\mathrm{A}\cdot\mathrm{m}) = 1\ \mathrm{kg\,A^{-1}s^{-2}}$.

The relation between the $\mathcal{E}_{ind}$ generated in the circuit and the rate of the change of the magnetic flux passing through the circuit is expressed by *Faraday's law*, according to which the electromotive force of induction is proportional to the rate of flux changes Φ

$$\mathcal{E}_{ind} = -\frac{\Delta\Phi}{\Delta t} . \qquad (1)$$

When dealing with a coil consisting of n series connected concentric turns, EMF is *n* times larger than in a single turn circuit:

$$\mathcal{E}_{ind} = -n\frac{\Delta\Phi}{\Delta t} . \qquad (2)$$



The described phenomenon of electromagnetic induction is used in transformers to increase or decrease (transform) the AC voltage. The transformer consists of primary and secondary windings wound on a common core made of mild steel, i.e. a material with high magnetic permeability. The core's purpose is to strengthen the magnetic flux going through the windings significantly. The core is made of metal sheets isolated from each other - such a construction hinders the creation of eddy currents in the core volume, which reduces energy losses due to the heating of the transformer.

The winding of a transformer connected to an AC source is called a primary winding; in turn the winding connected to an electricity receiver is called a secondary winding. When an alternating voltage is applied to the primary winding, an alternating current will flow through it, causing an alternating magnetic flux Φ in the core, which is almost completely concentrated inside the core and practically completely penetrates the secondary winding, inducing an induction EMF in it:

$$\mathcal{E}_2 = -n_2\frac{\Delta\Phi}{\Delta t} , \qquad (3)$$

where n_2 – number of turns in the secondary winding.

If a current I_2 flows in the secondary circuit with resistance R_2, the voltage at the ends of this winding is:

$$U_2 = \mathcal{E}_2 - I_2 R_2 . \qquad (4)$$

It causes the same flux of induction simultaneously in the primary winding (the number of turns n_1), EMF self-induction $\mathcal{E}_1$:

$$\mathcal{E}_1 = -n_1 \frac{\Delta\Phi}{\Delta t} . \qquad (5)$$

Then the voltage supplying the primary circuit satisfies the relationship:

$$U_1 = \mathcal{E}_1 + I_1 R_1 , \qquad (6)$$

R_1 is the resistance of the primary winding. Since, in general, the winding resistances are small, we can assumed that R_1 and R_2 approach zero ohm, then the voltages on the windings are equal to the electromotive forces:

$$U_1 = \mathcal{E}_1 = -n_1 \frac{\Delta\Phi}{\Delta t} \quad \text{and} \quad U_2 = \mathcal{E}_2 = -n_2 \frac{\Delta\Phi}{\Delta t} . \qquad (7)$$

Let's divide these equations (7) one by another and get:

$$U_2 = \frac{n_2}{n_1} U_1 . \qquad (8)$$

The ratio of the number of turns in both windings, equal to the ratio of the voltages, is called the transformer turns ratio k:

$$k = \frac{n_2}{n_1} = \frac{U_2}{U_1} \qquad (9)$$

The actual value of the ratio may differ from that calculated from relation (9) due to the finite ohmic resistance of the windings, core hysteresis, magnetic flux dissipation, etc. These factors also cause losses during energy transfer: heat generation in the windings, eddy currents and core magnetization.

The energy loss in the transformer is determined by a factor called the *transformer efficiency* η. Transformer efficiency η is the ratio of the current power in the secondary winding $P_2 = U_2 \cdot I_2$ to the power of the current in the primary winding $P_1 = U_1 \cdot I_1$:

$$\eta = \frac{P_2}{P_1} = \frac{U_2 \cdot I_2}{U_1 \cdot I_1} \qquad (10\ )$$

In modern transformers, the losses can be reduced to several percent. Considering that for such transformers the current powers in the primary and secondary windings are almost equal $U_1 \cdot I_1 = U_2 \cdot I_2$, we conclude that the currents in the primary and secondary windings are inversely proportional to the number of turns in these windings:

$$\frac{I_1}{I_2} = \frac{U_2}{U_1} = \frac{n_2}{n_1} = k .$$

Transformers have found widespread use in engineering and radio engineering because, by simply selecting the number of turns in the windings, we can change the alternating current voltage in any way we wish, limited only by the puncture resistance of the insulating materials.

Performance of the task

I. The Determination of the transformer ratio

1. Connect the circuit according to the scheme in Fig. 3:
   Z –power supply or autotransformer,
   V_1, V_2 – voltmeters

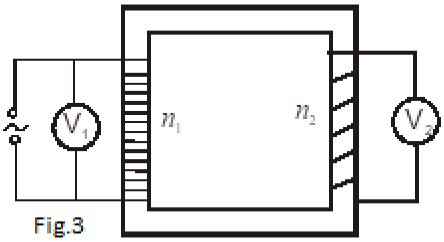


Fig.3

2. Set the voltage supplying the primary circuit to the value U_1 and read the voltage U_2.
3. Repeat the measurements for eight different U_1 voltage values. (e.g. close to 20V, 18V, 16V, etc.).
4. Calculate the ratio k_i for individual measurements and their average value — $\bar{k}$.

II. The determination of a transformer efficiency

1. Connect the circuit according to the diagram in fig. 4, R – resistor, A_1, A_2 – ammeters.

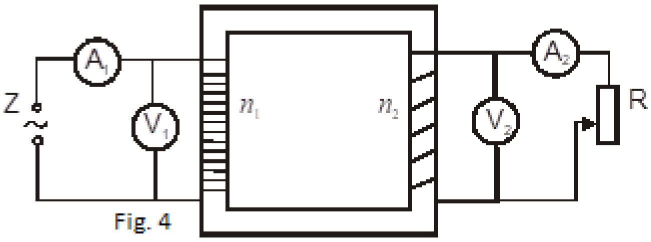


Fig. 4

2. Set the voltage of the power supply to a value close to 15 V, and at the lowest value of the adjustable resistor R, read the voltages and currents in the transformer windings.
3. Repeat the measurements several times, with increasing values of the resistance *R*.
4. Calculate an efficiency value for each measurement.
5. Draw a graph of the dependence of η on the power of the current in the secondary winding, $\eta = f(P_2)$.

WARNING. Before connecting the setup to the electric grid, set the potentiometer of the power supply to the zero value of the voltage, and the measuring instruments to the maximum measuring range. Once the measurement has started, the instrument ranges should be selected to use their most sensitive range. (e.g. the ammeter should display the value of 12.00 mA and not 0,012 A).

Calculation of the uncertainties

I. Transformer ratio: $k = U_2 / U_1$ **.**

We use the differential calculus to calculate the uncertainties of a single measurement of the transformer ratio Δk. Hence, we get the formula:

$$\Delta k = k\left(\frac{\Delta U_2}{U_2} + \frac{\Delta U_1}{U_1}\right).$$

II. The efficiency of the transformer (energy conversion efficiency): $\eta = \dfrac{U_2 \cdot I_2}{U_1 \cdot I_1}$.

All four quantities in the above formula are subjected to measurement errors. The absolute error of η is calculated using the differential calculus method. After the calculations, we get:

$$\Delta\eta = \eta\left(\frac{\Delta U_1}{U_1} + \frac{\Delta I_1}{I_1} + \frac{\Delta U_2}{U_2} + \frac{\Delta I_2}{I_2}\right).$$

III. The power of the current in the secondary winding: $P_2 = U_2 \cdot I_2$**.**

We also use the differential calculus for that calculation:

$$\Delta P_2 = P_2\left(\frac{\Delta U_2}{U_2} + \frac{\Delta I_2}{I_2}\right).$$

Absolute errors $\Delta\eta$ and ΔP_2 should be calculated for several measurements (round up the results) and mark on the chart $\eta = f(P_2)$.

The measurement accuracy of ΔU, ΔI were given in the instrument manual. In the case of digital meters used in the exercise, the accuracy (relative error) should be 1,5% for both AC voltage and current.