First name .... Date ....

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Exercise 242

Determination of the electrical resistance by the Wheatstone bridge method

Single resistors

Type of resistor		Length	Diameter	Cross-sectional area	Specific resistance
		l, [m]	Φ, [m]	S, [m^2]	[Ω·m]
a					ρ_a =
b					ρ_b =

Resistor	No Meas.	Reference resistance	Distance		Tested resistance	Average value
		R_w, [Ω]	l_1, [m]	l_2, [m]	R_x, [Ω]	R_x, [Ω]
a	0	10				R_a =
	1					
	2					
	3					
b	0	10				R_b =
	1					
	2					
	3					

Resistor combination

Type connections	Resistance standard	Distance		Resistance resultant
	R_w, [Ω]	l_1, [m]	l_2, [m]	[Ω]
series				R_s =
parallel				R_r =

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Introduction

According to Ohm's law, the current that flows through a conductor is proportional to the voltage applied to its ends. The ratio of the voltage U measured at the ends of the conductor to the current I is a constant value, called the *electric resistance R* of the conductor:

$$R = \frac{U}{I} \tag{1}$$

The ohm is the unit of electrical resistance [Ω]; 1 Ω = 1 V/A.

The electrical resistance of metallic conductors is the result of the interaction of current carriers (electrons) with the ions of the crystal lattice. It depends on the geometrical features of the conductor, i.e. the length l and the cross-sectional area S, and type of material it is made of:

$$R = \rho \frac{l}{S}. \tag{2}$$

The proportionality coefficient ρ characterizes the type of material is called the electrical resistivity (also specific electrical resistance or volume resistivity). The unit of specific resistance is 1 Ω·m (ohm-meter).

In electric circuits, there are combination of resistances connected in series (Fig. 1a) or parallel (Fig. 1b). The equivalent resistance R, in the case of n connected resistances, satisfies the formula:

- for a series connection

$$R = \sum_{i=1}^{n} R_i \,, \tag{3a}$$

- for a parallel connection

$$\frac{1}{R} = \sum_{i-1}^{n} \frac{1}{R_i}. \tag{3b}$$

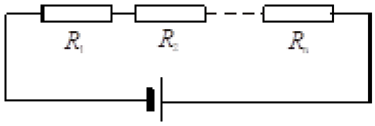

Fig. 1a

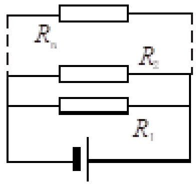

Fig. 1b

For two resistance ***a*** and ***b*** we get: $R = R_a + R_b$ (series connection),

$$\frac{1}{R} = \frac{1}{R_a} + \frac{1}{R_b} \text{ (parallel connection).}$$

These formulas are derived from Kirchhoff's laws, which describe the flow of current in electrical circuits.

Kirchhoff's first law (Kirchhoff's junction rule or nodal rule)

states that, for any node (junction) in an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of that node; or equivalently: the algebraic sum of currents in a network of conductors meeting at a point is zero:

$$\sum_{i=1}^{m} I_i = 0 \tag{4a}$$

Assumption: We consider the currents enering the node as positive, and the currents leaving the node as negative.

Kirchhoff's second law (Kirchhoff's loop rule)

states that, the directed sum of the potential differences (electromotive forces ε) around any closed loop is zero:

$$\sum_{k=1}^{n} \mathcal{E}_k = \sum_{j=1}^{m} R_j I_j \,. \tag{4b}$$

When applying this law, we choose the direction of determining the potentials ("circulation" direction) in the mesh (e.g. clockwise). For example, if the current source has a system of electrodes (from "-" to "+") following for assumed direction of circulation $\mathcal{E}_k$, then we assign a plus sign. Otherwise, we substitute $\mathcal{E}_k$ with a minus sign. The voltage drop across the resistance is positive ($R_i I_i$) when the current flows through the resistance R_i in the direction of circulation. The voltage drop is negative when the current flows in the opposite direction ($-R_i I_i$). If we obtain a negative current intensity from the calculations, it means that the real direction of the current is opposite to the assumed one.

Measurement of electrical resistance using a Wheatstone bridge

One of the simpler and more accurate methods of measuring resistance is the Wheatstone bridge circuit. Fig. 2 shows a diagram of the so-called linear Wheatstone bridge. We connect the determined resistance R_x in series with the reference resistance R_w (this is the resistance set on the decade resistor). Next, we link the free ends of the connected resistors with points A and C, between which a resistance wire is stretched along the millimetre scale. Point B, common to resistors R_x and R_w, is connected via the milliammeter G to slide rule D, which moves freely along the wire.

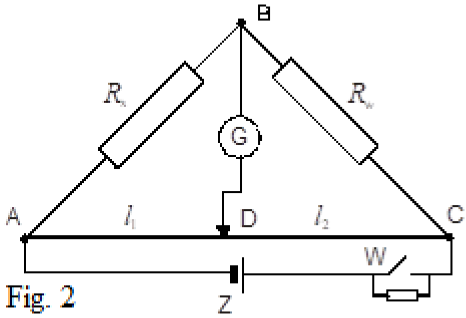

Fig. 2

Switching on the DC power supply, we apply voltage to points A and C. We are now looking for such a position of the slider D on the string that no current flows through section BD (the galvanometer should show zero). Then the bridge is balanced, i.e. the potentials of points B and D are equal, and the voltages in the various branches of the circuit satisfy the following conditions:

$$U_{AB} = U_{AD}, \quad U_{BC} = U_{DC}. \tag{5}$$

The first law of Kirchhoff shows that if there is no current flowing through the segment BD, then

$$I_{AB} = I_{BC}, \quad I_{AD} = I_{DC} \tag{6}$$

Using Ohm's law, we can express equalities (5) as follows:

$$R_x I_{AB} = R_{AD} I_{AD}, \quad R_w I_{BC} = R_{DC} I_{DC}.$$

After dividing these equations by sides and taking into account (6), we get:

$$R_x / R_w = R_{AD} / R_{DC}. \tag{7}$$

Due to the relationship(2), taking into account that the entire resistance wire has the same cross-sectional area everywhere, the resistances ratio R_{AD} / R_{DC} can be replaced by the ratio of their lengths:

$$R_{AD} / R_{DC} = l_1 / l_2. \tag{8}$$

From equations (7) and (8) we obtain the formula for the resistance value R_x, that we are looking for:

$$\boxed{R_x = \frac{l_1}{l_2} R_w}. \tag{9}$$

Performance of the task

1. Arrange the electric circuit as shown in Fig. 2, where: R_w — decade resistor, R_x — tested resistor, Z — DC source, G — galvanometer (milliammeter),W —circuit breaker with safety resistance.
2. With switch W open (protection resistor on), we set the decade resistor to 10 Ohm and switch on the Z supply (voltage about 2V). We choose such a position of the D slider on the string that the galvanometer shows a zero value. When the key W is closed, we set this position more precisely and read the lengths of the segments l_1 and l_2 .

3. Calculate the resistance value using the formula (9) - this is an approximate value, subject to a fairly large measurement error. Denote the determined value of R_x as R_0 .
4. From considerations based on error calculus, it follows that the minimum measurement error is obtained when the bridge is balanced with the slider set at half of the string length. Accordingly, appropriate measurements should be made by selecting the reference resistance R_w so that it is approximately equal to the value R_0 determined in point 3.
5. Proceeding as in point 2, three resistance measurements R_x should be taken - we set three different, but close to R_0, values of the refrence resistance on the decade resistor. In this way, we obtain three values of R_1, R_2, R_3. Take the arithmetic mean as the correct resistance value $R_a = (R_1 + R_2 + R_3)/3$.
6. In the same way, determine the unknown resistance of the next resistor — R_b.
7. Connect the tested resistors in series and measure the resistance with the use of Wheatstone bridge. Do the same for a parallel connection.

   Attention: For these measurements, as the reference resistance, we set the values that we calculate from formulas (3a) and (3b) to the equivalent resistance for series and parallel connections.
8. From the formula (2), we can calculate the specific resistances of the tested resistors.

Calculation of the uncertainties

Relative error of resistance measurement, $\Delta R_x / R_x$.

Calculate this error using differential calculus which we apply to the formula (9). Hence:

$$\frac{\Delta R_x}{R_x} = \frac{\Delta R_w}{R_w} + \frac{\Delta l_1}{l_1} + \frac{\Delta l_2}{l_2} . \qquad (10)$$

Absolute errors Δl_1 i Δl_2 are equal to the smallest value of the slider shift on the resistance wire for which there is a noticeable displacement of the galvanometer needle (these errors are not less than the accuracy of the length reading - 2 mm).

Value $\Delta R_w / R_w$ is determined by the class of the decade resistor. For example, a resistor class of 0.5 means that the relative error is equal 0.5%, which means $\frac{\Delta R_w}{R_w} = 0{,}005$.

Relative error of specific resistance, $\Delta\rho/\rho$.

Formula (2) shows that $\rho = \frac{SR_x}{l} = \frac{\pi\phi^2 R_x}{4l}$; here ϕ i l denotes the diameter and length of the wire from which the tested resistor was made. The differential calculus gives the following result:

$$\frac{\Delta\rho}{\rho} = \frac{\Delta R_x}{R_x} + 2\frac{\Delta\phi}{\phi} + \frac{\Delta l}{l} . \qquad (11)$$

Assumption: $\Delta\phi = 5 \cdot 10^{-6}$ m, $\Delta l = 2 \cdot 10^{-2}$ m.